ALGEBRA II-A
INTEGRAL EXPONENTS LAP
suggested Time: 1 week

Welcome to Algebra II. This is your first " LEARNING ACTIVITY PACKET", affectionately known as a L.A.P. or "lap". You will find if you read this carefully, and do all the problems herein, you will complete the unit successfully - with perhaps a little bit of help from your teacher and/or your fellow students.

This first L.A.P. won't contain too much that is new. You will have covered this material before Algebra I or even before that. But the review is good for you and will be necessary to strengthen your foundation for what will follow in this course. If you have missed any of this along the way, here is your chance to pick it up and make it part of your knowledge reportory.

I. Multiplying and dividing by powers of ten:

A. You must remember the "Short Cut" for multiplying and dividing by powers of ten. If not, study these examples:

$3.263 \times 10 = 32.63$

$15.65 \times 100 = 15.65 \times 10^2 = 1{,}565$

$.07263 \times 1{,}000 = .07263 \times 10^3 = 72.63$

$13 \times 1{,}000{,}000 = 13 \times 10^6 = 13{,}000{,}000$

$7.62 \div 10 = .762$

$213.73 \div 100 = 213.73 \div 10^2 = 2.1373$

$.024 \div 1{,}000 = .024 \div 10^3 = .000024$

$725 \div 1{,}000{,}000 = 725 \div 10^6 = .000725$

The rules:

To multiply by a power of ten, move the decimal point to the right - one place for each power of ten.

(The number of zeros in 10, 100, or 1,000, determines the number of places)

To divide by a power of ten, move the dicimal point to the left - one place for each power of ten

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B. Let's review the meaning of a negative exponent:

$$x^{-2} = \frac{1}{x^2}$$

(a negative says "take the reciprocal" and it becomes a positive exponent)

Now if we apply this to our examples above, look what happens:

$$3.2 \times 10^{-2} = 3.2 \times \frac{1}{10^2} = 3.2 \div 10^2 = .032$$

$$7.2 \div 10^{-4} = 7.2 \div \frac{1}{10^4} = 7.2 \times 10^4 = 72{,}000$$

∴ <u>multiplying by 10 to a negative power reverses the direction</u> in which you move the decimal point.

Assignment I.

Simplify these...

1. 32.53×10^3
2. 175.2×10^4
3. $.00726 \times 10^2$
4. $16.5 \div 10^2$
5. $173 \div 10^3$
6. $1{,}724 \times 10^2$
7. $.852 \times 10^4$
8. 37.24×10^{-2}
9. $.065 \div 10^{-3}$
10. 572×10^{-5}
11. 76.5×10^{-3}
12. $184 \div 10^{-2}$
13. $5{,}000 \times 10^3$
14. $.00726 \div 10^{-5}$
15. $52.9 \div 10^{-7}$
16. 65×10^3
17. $.00342 \times 10^5$
18. $.07 \times 10^5$
19. $37.2 \div 10^{-1}$
20. 52.63×10^{-3}

II. Scientific Notation

Scientific Notation is used as a simple way of writing very large or very small numbers. It is based on the premise that any number can be written as the <u>product</u> of a) a number between 1 and 10, and b) a power of 10

Some special cases can be written as "a" or "b" alone.

ex:

$1{,}000 = 10^3$
$1{,}000{,}000 = 10^6$
$.01 = 10^{-2}$
$.00001 = 10^{-5}$

} powers of 10

$7.632 = 7.632$
$5 = 5$
$1.62 = 1.62$

numbers between 1 and 10

Any other number must be expressed as a product of the two:

$7{,}000 = 7 \times 10^3$

$65.3 = 6.53 \times 10^{1}$
$184.32 = .8432 \times 10^{2}$
$.0763 = 7.63 \times 10^{-2}$
$.0004 = 4 \times 10^{-4}$

Assignment 2

Express these in scientific notation:

1. 826.5
2. 35.746
3. 5,000
4. 173,000
5. 7.32
6. 16.45
7. .0758
8. .964
9. 135.2
10. .00005
11. .0023
12. 10,000
13. 150,000
14. .0700
15. .00037
16. .243

III. Now let's review some properties of exponents: (you've learned these before.)

$X^a \cdot X^b = X^{a+b}$ ex. $a^2 \cdot a^3 = a^5$, or $2^3 \cdot 2^4 = 2^7$

$\frac{X^a}{X^b} = X^{a-b}$ ex. $\frac{a^7}{a^2} = a^5$

$(X^a)^b = X^{ab}$ ex. $(a^2)^4 = (a^2)(a^2)(a^2)(a^2) = a^8$

$(XY)^a = X^a Y^a$ ex. $(ab^2c^3)^4 = a^4b^8c^{12}$

$\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a}$ ex. $(2/5)^3 = \frac{8}{125}$

$X^{-a} = \frac{1}{X^a}$ ex. $10^{-3} = \frac{1}{10^3} = \frac{1}{1000}$

likewise, $\frac{1}{X^{-a}} = X^a$ ex. $\frac{1}{5^{-2}} = 5^2 = 25$

$\left(\frac{x}{y}\right)^{-a} = \frac{y^a}{x^a}$ ex. $\left(\frac{3}{a^2}\right)^{-3} = \left(\frac{a^2}{3}\right)^3 = \frac{(a^2)^3}{3^3} = \frac{a^6}{27}$

Applying these rules to some examples:

ex. $\frac{35a^3b^2c}{7ab^2c^5} = \left(\frac{35}{7}\right) \cdot \left(\frac{a^3}{a}\right) \cdot \left(\frac{b^2}{b^2}\right) \cdot \left(\frac{c}{c^5}\right) = 5 \cdot a^2 \cdot 1 \cdot \frac{1}{c^4} = \frac{5a^2}{c^4}$

ex. $\frac{a^{-2}}{a^{-5}} = \left(\frac{\frac{1}{a^2}}{\frac{1}{a^5}} = \frac{1}{a^2} \cdot \frac{a^5}{1}\right) = \frac{a^5}{a^2} = a^3$

optional method

(See note on next page)

NOTE: Always take care of the negative exponents ("take the reciprocal") BEFORE you do any other steps.

ex. $\frac{6a^{-2}b^3c}{12a^5b^{-2}c^{-3}} = \frac{6b^3 \cdot b^2c^3c}{a^2 \cdot 12a^5}$

$= \frac{6b^5c^4}{12a^7}$

$= \frac{b^5c^4}{2a^7}$

All we did in this first step was to get rid of the negative exponents - we "took the reciprocal" and made the exponents positive.

ex. $\frac{3^5}{3^{-3}} = 3^5 \cdot 3^3 = 3^8$ (Not 9^8 - the base stays the same as we add the exponents.)

ex. $\left(\frac{8a^2b^3}{2a^3b^5}\right)^2 = \left(\frac{4}{ab^2}\right)^2 = \frac{16}{a^2b^4}$

ex. $\frac{x^{w+2}}{x^2} = x^w$

ex. $\left(\frac{a^2b}{cd^4}\right)^{-2} = \left(\frac{cd^4}{a^2b}\right)^2 = \frac{c^2d^8}{a^4b^2}$

ex. $\frac{x^{-2}y^{-2}}{x^{-1}+y^{-1}} = \frac{1}{x^2y^2(\frac{1}{x} + \frac{1}{y})} = \frac{1}{\frac{x^2y^2}{x} + \frac{x^2y^2}{y}} = \frac{1}{xy^2 + x^2y}$

Some special cases to NOTE:

$(a+b)^2 = (a+b)(a+b) = a^2+2ab+b^2$

Note: $(a+b)^2 \neq a^2+b^2$

$-2^4 = -(2)(2)(2)(2) = -16$

$(-2)^4 = (-2)(-2)(-2)(-2) = +16$

Remember: an exponent acts only upon the factor immediately in front of it- unless parentheses are used.

ex. $7a^2 = 7 \cdot a^2$ $\quad (7a)^2 = 49a^2$

$-3^2 = -9$ $\quad (-3)^2 = 9$

$2a^{-2} = \frac{2}{a^2}$ $\quad (2a)^{-2} = \frac{1}{(2a)^2} = \frac{1}{4a^2}$

$7p^0 = 7(1) = 7$ $\quad (7p)^0 = 1$

SPECIAL NOTES ABOUT ZERO:

1. For every x not equal to zero, $x^0 = 1$
2. 0^0 is undefined
3. For every k greater than zero, $0^k = 0$
4. For every k less than zero, 0^k is undefined. This of course, would involve division by zero which is undefined.

Assignment III.

Simplify. Leave no negative exponents

1. $\frac{a^5b^3}{ab}$
2. $\frac{a^2}{a^9}$
3. $5^3 \cdot 5^4$
4. $a^2 \cdot a^3 \cdot a^0$
5. $w^5 \cdot w^{-3}$
6. $\frac{12a^3b^5}{4ab}$
7. $\frac{6a^2b^3}{3ab^5}$
8. $9a^0$
9. -2^6
10. $(5+2)^{-2}$
11. $\frac{a^{-3}}{a^{-8}}$
12. $c^a \cdot c^b \cdot c^{2-b}$
13. $\frac{x^5y^{-2}x}{x^3y^5z^{-3}}$
14. $\left(\frac{7a+2bxy}{5x^2y^3}\right)^0$
15. $(-3a)^2$
16. $(-5x^2y^3)^4$
17. $\frac{x^{w+5}}{x^w}$
18. $\left(\frac{9a^2}{3a^5}\right)^{-1}$
19. $\frac{b^{2a}}{b^a}$
20. $\frac{4x}{x^{-1}} + \frac{5}{x^{-2}}$
21. $\left(\frac{2a}{7x^2y^3}\right)^{-2}$
22. $(3x^2y)(x^3y^4)^{-3}$
23. $\frac{x^{-1}+y^{-2}}{x^{-2}y^{-3}}$
24. $\frac{7w^2y^{-2}z^3}{49w^5y^3z^{-2}}$
25. $\left(\frac{5ab}{3x^2y^3}\right)^{-2}$
26. $2^3 \cdot 2^4 \cdot 2^{-2}$
27. $\frac{5}{w^{-2}} \div w^5$
28. $7^0 + 5^0 + 2^0$
29. $a^k \cdot b^w \cdot a^{2k} \cdot b^3$
30. $\left(\frac{5wy}{2a}\right)^{-1}\left(\frac{6y}{2a^2}\right)^2$

Integral Exponents

IV' Multiplication of Polynomials

Look at these expamples:

1. $(a+2b)^2 = (a+2b)(a+2b) = a^2+4ab+4b^2$

2. $(a+b)^3 = \left[(a+b)(a+b)\right](a+b)$

$= (a^2+2ab+b^2)(a+b)$

$$\begin{array}{r} a^2+2ab+b^2 \\ \underline{\quad a+b \quad} \\ a^3+2a^2b+ab^2 \\ \underline{\quad a^2b+2ab^2+b^3 \quad} \\ a^3+3a^2b+3ab^2+b^3 \end{array}$$

3. $(w+3)(w^2-2)=w(w^2)+3(w^2)+w(-2)+3(-2)$

$=w^3+3w^2-2w-6$

4. $(x^a+3)(x^a+2)= x^{2a}+5x^a+6$

5. $5x^2y\left(x^3+\frac{4}{x^2y}-\frac{3}{x^5y^2}\right)$ DON'T FORGET THE DISTRIBUTIVE PROPERTY

$= 5x^5y+\frac{20x^2y}{x^2y}-\frac{15x^2y}{x^5y^2} = 5x^5y+20-\frac{15}{x^3y}$

Be sure you understand all the examples above before you try the next assignment. If you don't understand an example, ask a friend or your teacher.

Assignment 4.

1. $(5+w)^2$
2. $(3ab-2)^2$
3. $(x^w-3)^2$
4. $(t+5)(t^2-3)$
5. $(a-5)(a^2+9)$
6. $(a+5)^3$
7. $(w^x+2)^3$
8. $(a^{4n}+2)(a^m-1)$
9. $(b^w+3)(b^a+2)$
10. $3x^2(x^3-27x^2+\frac{2}{x^2})$
11. $-7a^2b(\frac{3}{a^5}+2a^3b^2-1)$
12. $25x^2y^3\left(\frac{1}{5}-\frac{3}{xy}+\frac{2}{x^2y^3}\right)$
13. $-12w^5\left(3w^{-2}-(2w)^2+\frac{1}{6w^7}\right)$
14. $(a+b)^4$
15. $(x+1)^4$

V. Division of Polynomials

When you deal with division, remember that you can only factor out factors, not addends. In other words, always factor the polynomials before you start to "cancel".

ex. $\frac{x^2-9}{x^2+6x+9}$ Don't " cancel" out the two x's. That is a "No-No".

First: Factor each polynomial:

$$\frac{x^2-9}{x^2+6x+9} = \frac{(x+3)(x-3)}{(x+3)(x+3)}$$

Now, simply as you would any numbers: (i.e. $\frac{35}{55} = \frac{5\cdot 7}{5\cdot 11} = \frac{5}{5}\cdot\frac{7}{11}$

$$= 1 \cdot \frac{7}{11} = \frac{7}{11}$$

therefore: $\frac{(x+3)(x-3)}{(x+3)(x+3)} = \frac{(x+3)(x-3)}{(x+3)(x+3)} = \frac{x-3}{x+3}$

ex. $\frac{x^2+5x-14}{x^2-49} = \frac{(x+7)(x-2)}{(x+7)(x-7)} = \frac{x-2}{x-7}$

ex. $\frac{x+2}{5} \div \frac{x^2+5x+6}{10} = \frac{x+2}{5} \cdot \frac{10}{x^2+5x+6} = \frac{x+2}{5} \cdot \frac{10^{2}}{(x+3)(x+2)} = \frac{2}{x+3}$

ex. $\left(\frac{2x^3}{5}\right)^{-1}\left(\frac{x^2}{6}\right)^{2} = \left(\frac{5}{2x^3}\right)^{1}\left(\frac{x^4}{36}\right) = \frac{5x^4}{72x^3} = \frac{5x}{72}$

ex. $(a^2+a-12)(a^2-4a+3)^{-1}$

$$= \frac{(a^2+a-12)}{(a^2-4a+3)} = \frac{(a+4)(a-3)}{(a-3)(a-1)} = \frac{a+4}{a-1}$$

Assignment 5.

5. $\frac{12x^3}{5}\left(\frac{10}{x^3}\right)^3$

Simplify:

1. $\frac{a^2-25}{a^2+10a+25}$

2. $\frac{x^2+x-6}{x^2+3x}$

3. $\frac{x-6}{x^2-4} \div \frac{2x-12}{x+2}$

4. $(2a^2)(3b)^{-1}(15b^3)(2a)^{-2}$

6. $\frac{5a^3}{2} \div \frac{a^2}{3}$

7. $\frac{6abc^2}{15a^2}\left(\frac{2bc}{3a}\right)^{-1}$

8. $\frac{x^2+3x}{x^2+2x-3} \cdot \frac{x-1}{x}$

9. $\frac{y^2+6y-16}{y^2-64}(y-2)^{-1}$

10. $\dfrac{2a-8}{a^2-4} \div \dfrac{a-4}{a^2+6a+8}$

11. $\dfrac{3a+6c}{9a} \cdot \dfrac{12ac}{a^2-4c^2} \div \dfrac{18a^3c^3}{2a-4c}$

12. $\left(\dfrac{5x^2}{8}\right)^{-1} \left(\dfrac{x}{2}\right)^{-3}$ 13. $\dfrac{(a-b)^2}{a^2-b^2}$ 14. $\left(\dfrac{10-5x}{3x^2}\right)\left(\dfrac{2-x}{12x^3}\right)^{-1}$

That's it folks !! Now check all your answers and find out why the incorrect ones are wrong. Do this before you take the "Trial Run". Your friends can help you here or ask your teacher.

After you understand all your errors, it is time to take the "Trial Run". This is just like the test - take it as you would take a test. But then you check your answers to see how you did. That will tell you whether or not you're ready to take the REAL test. (If not, get help !)

Good Luck!!

ANSWERS TO LAP

assignment 1.

1. 32,530
2. 1,752,000
3. .726
4. .165
5. .173
6. 172,400
7. 8,520
8. .3724
9. 65
10. .00572
11. .0765
12. 18,400
13. 5,000,000
14. 726
15. 529,000,000
16. 65,000
17. 342
18. 7,000
19. 372
20. .05263

assignment 2

1. 8.265×10^2
2. 3.5726×10^1
3. 5×10^3
4. 1.73×10^5
5. 7.32
6. 1.645×10
7. 7.58×10^{-2}
8. 9.64×10^{-1}
9. 1.352×10^2
10. 5×10^{-5}
11. 2.3×10^{-3}
12. 10^4
13. 1.5×10^5
14. 7×10^{-2}
15. 3.7×10^{-4}
16. 2.43×10^{-1}

INTEGRAL EXPONENTS

Answers to Lap

Assinment 3

1. a^4b^2
2. $\frac{1}{a^7}$
3. 5^7
4. a^5
5. w^2
6. $3a^2b^4$
7. $\frac{2a}{b^2}$
8. 9
9. -64
10. 1/49
11. a^5
12. c^{a+2}
13. $\frac{x^2z^4}{y^7}$
14. 1
15. $9a^2$
16. $625x^8y^{12}$
17. x^5
18. $\frac{a^3}{3}$
19. b^a
20. $9x^2$
21. $\frac{49x^4y^6}{4a^2}$
22. $\frac{3}{x^7y^{11}}$
23. xy^3+x^2y
24. $\frac{z^6}{7w^3y^5}$
25. $\frac{9x^4y^6}{25a^2b^2}$
26. $2^5=32$
27. $\frac{5}{w^3}$
28. 3
29. $a^{3k}b^{w+3}$
30. $\frac{18y}{5wa^3}$

assignment 4.

1. $25+10w+w^2$
2. $9a^2B^2-12ab+4$
3. $x^{2w}-6x^w+9$
4. $t^3+5t^2-3t-15$
5. $a^3-5a^2+9a-45$
6. $a^3+15a^2+75a+125$
7. $w^{3x}+6w^{2x}+12w^x+8$
8. $a^{5n}-a^{4n}+2a^n-2$
9. $b^{w+a}+2b^w+3b^q+6$
10. $3x^5-81x^4+6/x^3$
11. $\frac{-21b}{a^3}-14a^5b^3+7a^2b$
12. $5x^2y^3-75xy^2+50$
13. $-36w^3+48w^7-2/w^2$
14. $a^4+4a^3b+6a^2b^2+4ab^3+b^4$
15. $x^4+4x^3+6x^2+4x+1$

assignment 5.

1. 1. $\frac{a-5}{a+5}$
2. $\frac{x-2}{x}$
3. $\frac{1}{2(x-2)}$ or $\frac{1}{2x-4}$
4. $5b^2/2$
5. $\frac{2400}{x^6}$
6. 15a/2
7. $\frac{3c}{5}$
8. 1
9. $\frac{1}{y-8}$
10. $\frac{2(a+4)}{a-2}$ or $\frac{2a+8}{a-2}$
11. $\frac{4}{9a^3c^2}$
12. x/5
13. $\frac{a-b}{a+b}$
14. 20x

TRIAL RUN

I. Express these numbers using scientific notation:

A. 300,000,000 B. .000077 C. 54.200 D. .03200

II. Express these numbers without using exponents:

A. 2.2×10^{-8} B. 4.35×10^{2} C. 4.5×10^{-3} D. 6.57×10^{5}

III. Simplify. Leave no negative exponents:

1. $2^0 + 5^0$ 2. $\left(\frac{1}{2}\right)^{-3}$ 3. $(-2x)^3 y^{-5}$

4. $\frac{x^{-5}y^2z^4}{x^3y^3z^{-1}}$ 5. $a^{10x} a^{-2x} a^{x}$ 6. $y^{k+1} z^{k-2} y^{k} z^{2}$

7. $\left(\frac{2ab}{3a^{-2}}\right)^3$ 8. $\left(\frac{3x^2}{8z^3}\right)^{-1}\left(\frac{6x^3}{2z}\right)^2$ 9. $(-4)^2$

10. -4^2 11. $\frac{6}{a^{-2}} \div a^3$ 12. $\frac{a^{-1}b^{-1}}{a^{-1} + b^{-1}}$

IV. Expand and simplify:

1. $(a - 3)^2$ 2. $(x^{2a} + 1)(x^{a} - 1)$

3. $(x + 2)(x^2 - 4)$ 4. $(a + 2)^3$

5. $3x^4(2x^{-4} + \frac{x^2}{6} - \frac{1}{x^3})$

V. Simplify:

1. $\frac{a^2 - 4}{(a + 2)^2}$ 2. $\frac{x - 1}{8} \div \frac{x^2 + 2x - 3}{4}$

3. $\left(\frac{3x^2}{8}\right)^{-1}\left(\frac{x}{2}\right)^3$ 4. $(x^2 + 7x + 12)(x^2 - 16)^{-1}$

INTEGRAL EXPONENTS — TRIAL RUN ANSWERS

I. A. 3.0×10^{8} B. 7.7×10^{-5} C. 5.42×10 D. 3.2×10^{-2}

II. A. 0.000000022 B. 435 C. 0.0045 D. 657000

III. 1. 2 2. 8 3. $\frac{-8x^3}{y^5}$ 4. $\frac{z^5}{x^8 y}$

5. a^{9x} 6. $y^{2k+1} z^{k}$ 7. $\frac{8 a^9 b^3}{27}$ 8. $24 x^4 z$

9. 16 10. -16 11. $\frac{6}{a}$ 12. $\frac{1}{b + a}$

IV. 1. $a^2 - 6a + 9$ 2. $x^{3a} - x^{2a} + x^{a} - 1$

3. $x^3 + 2x^2 - 4x - 8$ 4. $a^3 + 6a^2 + 12a + 8$

5. $6 + \frac{x^6}{2} - 3x$

V. 1. $\frac{a - 2}{a + 2}$ 2. $\frac{1}{2(x + 3)}$ OR $\frac{1}{2x + 6}$ 3. $\frac{x}{3}$

4. $\frac{x + 3}{x - 4}$