LEARNING ACTIVITIES PACKAGE

H-ALGEBRA 2

COMPLEX NUMBERS

...VIORAL OBJECTIVES

I. Define

A. The imaginary unit
B. A pure imaginary number
C. A complex number
D. Equality for two complex numbers
E. The conjugate of a complex number
F. The absolute value of a complex number

* II. Given i^k where k is any integer, change i^k to its equivalent; i.e.: i, -1, -i, or 1.

III. Given two pure imaginary numbers determine

A. Their sum
B. Their difference
C. Their product
D. Their quotient

IV. Given a complex number a + bi, determine

A. Its conjugate
B. Its absolute value
C. Its position on the complex number plane
* D. $(a + bi)^k$ where k is any positive integer

V. Write any given complex number in standard form

VI Given two complex numbers, be able to write in standard form

A. Their sum
B. Their difference
C. Their product
D. Their quotient

* VII. Solve equations involving the definition of equality of two complex numbers

VIII. Determine the nature of the roots of quadratic equations using the discriminanent

IX. Solve quadratic equations over the set of complex numbers.

SECTION I

THE IMAGINARY UNIT

Within the limits of the real number system, can you solve the following?

$$x^2 + 1 = 0$$

Our algebra techniques tell us to add -1 to both sides of the equation. Thus we would have:

$$x^2 = -1$$

At this point we would have to say, "No way!"

This L.A.P. introduces us to a new set of numbers which is expanded to include the solution for the above example.

We create a new number; i. **i is the imaginary unit**

$$i^2 = -1$$

$$\boxed{i = \sqrt{-1}}$$

A pure imaginary number is the square root of a negative real number.

An imaginary number can be written as the product of a real number and i; $(\sqrt{-1})$

Examples:

1. $\sqrt{-1} = i$
2. $\sqrt{-5} = \sqrt{-1}\ \sqrt{5} = i\sqrt{5}$
3. $\sqrt{-9} = \sqrt{-1}\ \sqrt{9} = i\sqrt{9} = 3i$
4. $\sqrt{-125} = \sqrt{-1}\ \sqrt{25}\ \sqrt{5} = i \cdot 5\sqrt{5} = 5i\sqrt{5}$

Remember to simplify—as you did with square roots when you worked in the set of real numbers.

Note: Let us stop for a minute and consider the name we are calling these "new" numbers. They are not called "imaginary" because they do not exist, but rather as opposed to the other numbers with which we have worked for so long, which were called real numbers. The name imaginary reflects the feeling on the part of some mathematicians who, many years ago, felt that such numbers did not exist. Today these numbers are useful in many branches of science. Still, the name "imaginary" remains.

EXERCISE 1

Simplify:

1. $\sqrt{-1}$
2. $\sqrt{-7}$
3. $2\sqrt{-6}$
4. $\sqrt{-12}$
5. $-\sqrt{-11}$
6. $5\sqrt{-27}$
7. $3\sqrt{-24}$
8. $\sqrt{-121}$
9. $\sqrt{-300}$
10. $\sqrt{\frac{-4}{9}}$
11. $-\sqrt{-72}$
12. $-\sqrt{-64}$

SECTION II OPERATIONS WITH IMAGINARY NUMBERS

POWERS OF IMAGINARY NUMBERS

A. By expressing pure imaginary numbers in i form, we can easily add, subtract, multiply and divide imaginary numbers in the same way as we do with real numbers.

Example 1: Add $\sqrt{-4}$ and $\sqrt{-9}$

Solution: $\sqrt{-4} + \sqrt{-9} = i\sqrt{4} + i\sqrt{9}$
$= 2i + 3i$
$= 5i$

Example 2: Subtract $\sqrt{-18}$ from $\sqrt{-8}$

Solution: $\sqrt{-8} - \sqrt{-18} = i\sqrt{4}\sqrt{2} - i\sqrt{9}\sqrt{2}$
$= 2i\sqrt{2} - 3i\sqrt{2}$
$= (2i - 3i)\sqrt{2}$
$= -i\sqrt{2}$

Example 3: Multiply $\sqrt{-5}$ by $\sqrt{-3}$

Solution: $(\sqrt{-5})(\sqrt{-3}) = (i\sqrt{5})(i\sqrt{3})$
$= i \cdot i\sqrt{5}\sqrt{3}$
$= i^2\sqrt{15}$
$= -1(\sqrt{15})$
$(\sqrt{-5})(\sqrt{-2}) = -\sqrt{15}$

Note: You must use the i form in this case to arrive at the at the correct answer. If you do not use the i form initally, you would have $\sqrt{-5}\sqrt{-3} = \sqrt{(-5)(-3)} = \sqrt{15}$. Don't do it! Mathematicians have agreed to call $-\sqrt{15}$ the correct solution in these cases.

Example 4: Divide $\sqrt{-15}$ by $\sqrt{-3}$

Solution: $\sqrt{-15} \div \sqrt{-3} = i\sqrt{15} \div i\sqrt{3}$
$= \dfrac{i\sqrt{15}}{i\sqrt{3}}$
$= \dfrac{i}{i} \cdot \dfrac{\sqrt{15}}{\sqrt{3}}$
$= \sqrt{\dfrac{15}{3}}$

$\sqrt{-15} \div \sqrt{-3} = \sqrt{5}$

Example 5: Divide $\sqrt{-2}$ by $\sqrt{-5}$

Solution: $\sqrt{-2} \div \sqrt{-5} = i\sqrt{2} \div i\sqrt{5}$

$$= \frac{i\sqrt{2}}{i\sqrt{5}}$$

$$= \frac{\sqrt{2}}{\sqrt{5}}$$

$$= \frac{\sqrt{2}\sqrt{5}}{\sqrt{5}\sqrt{5}}$$ (Remember...RATIONALIZE THE DENOMINATOR)

$$\sqrt{-2} \div \sqrt{-5} = \frac{\sqrt{10}}{5}$$

B. i^k where k is an integer form an interesting pattern. Note the following:

1. i; 2. $i^2 = i \cdot i = -1$; 3. $i^3 = i^2 \cdot i = -i$; 4. $i^4 = i^2 \cdot i^2 = 1$

STOP! MEMORIZE THE ABOVE FOUR EXAMPLES. WE REPEAT, MEMORIZE THE ABOVE FOUR EXAMPLES! ! !

i^k, for k an integer will simplify to i, -1, $-i$, or 1.

Examples:

1. $i^5 = i^4 \cdot i = 1 \cdot i = i$
2. $i^7 = i^4 \cdot i^3 = 1 \cdot -i = -i$
3. $i^{12} = i^4 \cdot i^4 \cdot i^4 = 1 \cdot 1 \cdot 1 = 1$
4. $i^{22} = (i^4)^5 \cdot i^2 = -1$

Since $i^4 = 1$, and 1 is the multiplicative identity, it follows that all integral powers of i^4 can be factored out and ignored. For example 4 above, i^{22}, we could have simply taken out all the 4's and concentrated on what was left, namely i^2.

Examples:

5. $i^{43} = ?$ $43 \div 4 = 10$ with a remainder of 3. Hence, we have ten i^4's and one i^3. $i^3 = -i$. $-i$ is the simplification of i^{43}.

6. $i^{61} = ?$ $61 \div 4 = 15$ with a remainder of 1. $i^1 = i$. i is the simplification of i^{61}.

7. $i^{720} = ?$ $720 \div 4 = 180$ with no remainder. Hence, we have only a product of i^4's. Since $i^4 = 1$, the simplification of i^{720} is 1.

ASSIGNMENT # 2

A. Perform the indicated operations:

1. $\sqrt{-4} + \sqrt{-16} - \sqrt{-25}$
2. $\sqrt{-4} + \sqrt{-9} + \sqrt{-25}$
3. $4\sqrt{-3} - \sqrt{-3}$
4. $\sqrt{-20} + \sqrt{-8}$
5. $\sqrt{-3} + \sqrt{-27}$
6. $\sqrt{-20} + \sqrt{-12}$
7. $\sqrt{-2} + \sqrt{-8}$
8. $\sqrt{-98} - \sqrt{-50}$
9. $2\sqrt{-12} - 2\sqrt{-48}$
10. $\sqrt{-\frac{3}{4}} + \sqrt{-\frac{1}{3}}$
11. $i^2 + i + i^3 + i^4$
12. $i^5 + i^6 + i^7 + i^8$
13. $\sqrt{-3} \cdot \sqrt{-7}$
14. $\sqrt{-3} \cdot \sqrt{-5}$
15. $\sqrt{-16} \cdot \sqrt{-9}$
16. $(\sqrt{-3})^2$
17. $\sqrt[3]{-8} \cdot \sqrt{-4}$
18. $\sqrt{-4} \cdot \sqrt[3]{8}$
19. $\sqrt{-4} \cdot \sqrt{-5}$
20. $3i \cdot 5i \cdot 2i$
21. $\frac{\sqrt{-6}}{\sqrt{-2}}$
22. $\frac{\sqrt{-15}}{\sqrt{-5}}$
23. $\frac{\sqrt{-2}}{\sqrt{-8}}$
24. $\frac{-5\sqrt{-3}}{2\sqrt{-6}}$
25. $\frac{\sqrt{-10}}{2\sqrt{-5}}$

B. Simplify

1. i^3
2. i^8
3. i^{17}
4. i^{38}
5. i^{200}
6. i^{723}
7. $i^{1,000}$
8. i^{1233}
9. $(i^{41})^3$

C. Find the complete solution set from the domain of positive integers. $k \in I^+$

1. $i^k = 1$
2. $i^k = i$
3. $i^k = -1$
4. $i^k = -i$

SECTION III THE SET OF COMPLEX NUMBERS

We can now expand our knowledge of numbers to include a new set of numbers: a number which consists of a real number and an imaginary number is a COMPLEX NUMBER. We write it in the standard form $a + bi$ where a and b are real numbers and $i = \sqrt{-1}$.

Examples:

1. $4 + 3i$
2. $17 - 32i$
3. $7 + 0\,i$ (When $b = 0$, the complex number is also a real number. Every real number, therefore, is a complex number, where $b = 0$. The set of real numbers is a subset of the set of complex numbers.)

4\. $0 + 3i = 3i$ (When $a = 0$, the complex number is also a pure imaginary number. The imaginary numbers are a subset of the complex numbers.)

Assignment #3

1\. Express each of these complex numbers in standard form: $a + bi$. Give the values of a and b for each.

A. $3 + 2i$ B. $-5 + 6i$ C. 17 D. $-3i$

E. $5 - i$ F. $\frac{4 + 2i}{2}$ G. $8 + \sqrt{-5}$ H. $5(3 - 4i)$

I. $8 - \sqrt{-5}$ J. $i^3 + i^4$

2\. Under what condition is $a + bi$ a real number?

3\. Under what condition is $a + bi$ a pure imaginary number?

SECTION IV EQUALITY

Two complex numbers are equal if and only if their real parts are equal and their imaginary parts are equal.

Example: $4 + 7i = x + yi$ if and only if $x = 4$ and $y = 7$

Assignment #4

Determine the real numbers x and y for which the equation is true

A. $x + yi = 7 + 3i$ B. $x + yi = -2 + 5i$

C. $x + yi + 5 = 13 + i - 5i$ D. $x + yi + 17 = 12$

E. $x + yi = 19i$ F. $(x + yi) + (7 - 3i) = 5 + 2i$ G. $2x + yi = 1 + (3+7i)$

SECTION V COMPLEX NUMBERS--ADDITION, SUBTRACTION, MULTIPLICATION, DIVISION

A. ADDITION: To find the sum of two complex numbers, add the real parts and add the imaginary parts.

Example: $(5 + 7i) + (8 + 2i) = 13 + 9i$

B. SUBTRACTION: To find the difference between two complex numbers, subtract the real parts and subtract the imaginary parts.

Example: $(5 + 2i) - (2 + 8i) = 3 - 6i$; In standard form: $3 + {}^{-}6i$

C. MULTIPLICATION: To find the product of two complex numbers, multiply as you would any two binomials. Then simplify.

Example:
$$\begin{aligned}(7 + 2i)(3 - 5i) &= 21 + 6i - 35i - 10i^2\\ &= 21 - 29i - 10(-1)\\ &= 21 - 29i + 10\\ &= 31 - 29i\end{aligned}$$

D. DIVISION: Division of two complex numbers is the most challenging of the operations. The process is one of rationalizing the denominator. Simply multiply the numerator and the denominator of the given expression by the conjugate of the denominator.

DEFINITION: The CONJUGATE of a complex number is another complex number different only in sign of the imaginary part from the given complex number.

Example: The conjugate of $3 + 2i$ is $3 - 2i$

The notation denoting the conjugate is a horizontal bar over the given number.

Examples: $\overline{7 + 2i} = 7 - 2i$

$\overline{-8 - 7i} = -8 + 7i$

$\overline{7} = 7$

$\overline{5i} = -5i$

Now for division

Example: $\frac{3 + 5i}{2 - 5i} \cdot (\quad\quad)$ ← [Multiply by the multiplicative identity 1, calling it N/N, where N = the conjugate of the denominator.]

So: $\frac{3 + 5i}{2 - 5i} \cdot \frac{2 + 5i}{2 + 5i} = \frac{6 + 10i + 15i + 25i^2}{4 - 25i^2}$

$= \frac{6 + 25i + 25(-1)}{4 - 25(-1)}$

$= \frac{-19 + 25i}{29}$ [NO SQUARE ROOTS IN THE DENOMINATOR]

ASSIGNMENT # 5

A. For each of the exercises #1 - 12, let r be the first complex number and s be the second. Compute: a) r + s; b) r - s; c) rs; and d) r/s

	r	s		r	s
1.	$2 + 9i$	i	7.	$5 + 6i$	$6i - 5$
2.	$15 + 4i$	$-i$	8.	$7i - 1$	$1 + 7i$
3.	$-i$	$1 + 3i$	9.	$-2 + 6i$	$3 + 4i$
4.	$2i$	$1 - i$	10.	$3 + i$	$3 + 7i$
5.	7	$5 - 2i$	11.	$1 + \sqrt{-25}$	$\sqrt{-36} - 2$
6.	-6	$-1 + 7i$	12.	$\sqrt{-49} - 3$	$-\sqrt{-1}$

B. Determine the complex number, a + bi, equal to each of the following:

13. $(7 + 8i) + (2 - 4i) - (3 - 8i)$
14. $(-4 + 2i) - (6 + 8i) + (13 - 2i)$
15. $-(9 - 4i) + (5 + i) - (-1)$
16. $-(7 + 3i) - (1 - 6i) - 3i$

C. Find the complex number indicated:

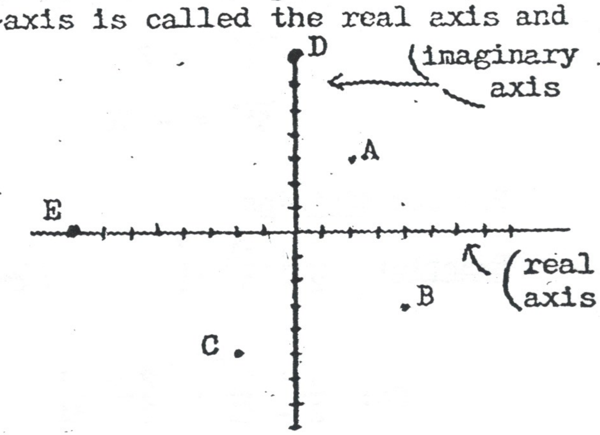
17. $\overline{4 + 3i}$
18. $\overline{-2 + 6i}$
19. $\overline{4 + \sqrt{3}\, i}$
20. $\overline{3}$
21. $\overline{\sqrt{5} - i}$
22. $\overline{8 - 4i}$
23. $\overline{-4i}$
24. $\overline{\sqrt{7}}$
25. $\overline{a + bi}$
26. $\overline{x - yi}$
27. $\overline{x + yi}$ is the ________ of x + yi.

SECTION VI GRAPHING COMPLEX NUMBERS

The complex number a + bi is sometimes represented by the ordered pair (a,b). The rectangular Cartesian coordinate system can be used to represent complex numbers graphically. For this purpose, the X-axis is called the real axis and the Y-axis is called the imaginary axis.

Examples:

1. A: $2 + 3i \rightarrow (2,3)$
2. B: $4 - 3i \rightarrow (4,-3)$
3. C: $-2 - 5i \rightarrow (-2,-5)$
4. D: $7i \rightarrow (0,7)$
5. E: $-8 \rightarrow (-8,0)$



ASSIGNMENT #6

Graph each of the following complex numbers on a complex coordinate system

A: $3 + 5i$ B: $-2 + 6i$ C: $8 - 3i$ D: $-6 - i$

E: $12i$ F: -8 G: $1 + i$ H. $-4i$

2. The vertical axis is called the ________ axis.
3. The horizontal axis is called the ________ axis.
4. On what axis would $-6i$ be plotted?
5. On what axis would 15 be plotted?

SECTION VII THE ABSOLUTE VALUE OF A COMPLEX NUMBER

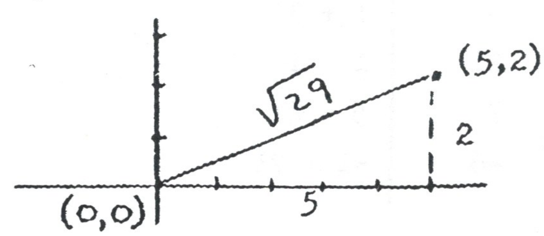
The absolute value of a complex number is the measure of the distance from the graph of that number to the point (0,0).

Example: 1. $|5 + 2i|$

A. Graph the number.

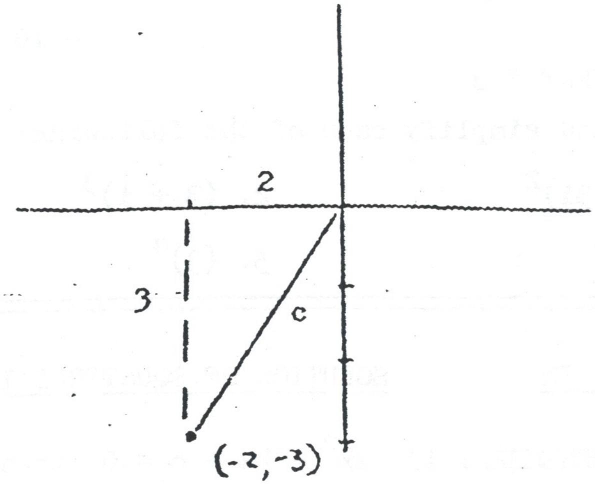
B. Find the length of the segment from (5,2) to (0,0).

C. The Pythagorean theorem allows us to determine that the distance is $\sqrt{29}$.



Example 2: $-2 - 3i$

Graph: $(-2, -3)$



Pythagorean theorem:

$$c^2 = (2)^2 + (3)^2$$
$$= 2^2 + 3^2$$
$$c = \sqrt{13}$$
$$|-2 - 3i| = \sqrt{13}$$

It is not necessary to graph a complex number in order to find its absolute value. We generalize the following:

$$\boxed{|a + bi| = \sqrt{a^2 + b^2}}$$

Since a and b are each squared, note that:

$$|a + bi| = |-a + bi| = |a - bi| = |-a - bi|$$

Examples using the formula:

1. $|-2 + 7i| = \sqrt{4 + 49}$
   $= \sqrt{53}$

2. $|-2 - 6i| = \sqrt{4 + 36}$
   $= \sqrt{40}$
   $= 2\sqrt{10}$

3. $|7i| = \sqrt{0 + 49}$
   $= \sqrt{49}$
   $= 7$

ASSIGNMENT # 7

Evaluate each of the following:

1. $|2 + 5i|$ 2. $|-3 - 9i|$ 3. $|6 - 8i|$ 4. $|17|$

5. $|-5i|$ 6. $|2 - 2i|$ 7. $|0|$ 8. $|12 - i|$ 9. $|5 + i|$

SECTION VIII * POWERS OF COMPLEX NUMBERS

Study the examples below:

Example 1: $(3 + 4i)^2 = (3 + 4i)(3 + 4i)$
$$= 9 + 12i + 12i + 16i^2$$
$$= 9 + 24i - 16$$
$$= -7 + 24i$$

Example 2: $(2 + 7i)^3 = (2 + 7i)(2 + 7i)(2 + 7i)$
$$= (4 + 28i - 49)(2 + 7i)$$
$$= (-45 + 28i)(2 + 7i)$$
$$= -90 - 259i - 196$$
$$= -289 - 259i$$

NOTE: THE COMPLEX NUMBERS ARE CLOSED UNDER THE OPERATION OF MULTIPLICATION. THE PRODUCT IS ANOTHER COMPLEX NUMBER.

Put it in the form
$a + bi$.

ASSIGNMENT # 8

xpand and simplify each of the following:

. $(5 + 3i)^2$ 2. $(3 + i)^3$ 3. $(2 - i)^3$

. $(5i)^4$ 5. $(3)^4$ 6. $(5 + 3i)^4$

ECTION IX SOLUTION OF EQUATIONS INVOLVING COMPLEX NUMBERS

REMEMBER: If $ax^2 + bx + c = 0$ then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

t is good to see this friend again! The quadratic formula is used to solve uadratic equations.

Recall: For the discriminant: $b^2 - 4ac > 0 \rightarrow$ Two real roots

$b^2 - 4ac = 0 \rightarrow$ One real root

$b^2 - 4ac < 0 \rightarrow$ No real roots.

Now with our knowledge of the number system expanded, we judge that when the discriminant is less than zero there are two complex roots.

AGAIN: $b^2 - 4ac < 0 \rightarrow$ TWO COMPLEX ROOTS

The discriminant can be used to determine the nature of the roots of a quadratic equation without actually solving the equation. Evaluate $b^2 - 4ac$ and then make the proper judgement as to the nature of the roots; i.e. two real, one real, or two complex.

Example 1: Solve $2x^2 + 3x + 5 = 0$

Solution: $a = 2$; $b = 3$; $c = 5$

$$x = \frac{-3 \pm \sqrt{9 - 40}}{4}$$

$$x = \frac{-3 \pm \sqrt{-31}}{4}$$

$$x = \frac{-3 \pm i\sqrt{31}}{4}$$

NOTE: This equation has two complex roots.

The two roots are conjugates of each other

-mple 2: $3x^2 = -2x - 5$

Solution: First put in standard form: $3x^2 + 2x + 5 = 0$

$a = 3, \quad b = 2, \quad c = 5$

$$x = \frac{-2 \pm \sqrt{4 - 60}}{6}$$

$$x = \frac{-2 \pm \sqrt{-56}}{6}$$

$$x = \frac{-2 \pm 2i\sqrt{14}}{6}$$

$$x = \frac{2(-1 \pm i\sqrt{14})}{6}$$

$$x = \frac{-1 \pm i\sqrt{14}}{3}$$

Again, we get two complex roots. The roots are conjugates of each other. Complex roots are always conjugates.

SSIGNMENT #9

Without solving the equation, determine the nature of the roots of each of the llowing:

a. $x^2 + 4x - 5 = 0$ b. $x^2 - 6x - 7 = 0$ c. $-3t^2 + 4t + 1 = 0$

d. $-5t^2 - t - 3 = 0$ e. $-7s^2 + 9s - 3 = 0$ f. $-4s + 3s^2 = -\frac{4}{3}$

g. $\sqrt{5}\,a^2 = 6a - \sqrt{5}$ h. $x^2 + \frac{7}{4} = \sqrt{7}\,x$ i. $3x^2 + 7 = 0$

j. $\sqrt{2}\,v^2 - v = 0$

:. Solve each of the equations below over the set of complex numbers:

a. $a^2 - 9a + 18 = 0$ b. $x^2 + x + 1 = 0$ c. $w^2 + w + 3 = 0$

d. $b^2 + 5b - 3 = 0$ e. $2x^2 = 5x - 7$ f. $\frac{t^2}{2} + \frac{3t}{2} = \frac{3}{10}$

g. $\frac{a}{2} + \frac{2}{3} = \frac{a^2}{6}$

HEY GANG, --- --- NOW IT'S TIME FOR THE TRIAL RUN!

L.A.P. ANSWERS

ASSIGNMENT #1

1. i 2. $i\sqrt{7}$ 3. $2i\sqrt{6}$ 4. $2i\sqrt{3}$ 5. $-i\sqrt{11}$ 6. $15i\sqrt{3}$
7. $6i\sqrt{6}$ 8. $11i$ 9. $10i\sqrt{3}$ 10. $\frac{2}{3}i$ 11. $-6i\sqrt{2}$ 12. $-8i$

ASSIGNMENT #2

A. 1. i 2. $10i$ 3. $3i\sqrt{3}$ 4. $2i(\sqrt{5}+\sqrt{2})$ 5. $4i\sqrt{3}$ 6. $2i(\sqrt{5}+\sqrt{3})$
7. $3i\sqrt{2}$ 8. $2i\sqrt{2}$ 9. $-4i\sqrt{3}$ 10. $\frac{5i\sqrt{3}}{6}$ 11. 0 12. 0
13. $-\sqrt{21}$ 14. $-\sqrt{15}$ 15. -12 16. -3 17. $-4i$ 18. $4i$
19. $-2\sqrt{5}$ 20. $-30i$ 21. $\sqrt{3}$ 22. $\sqrt{3}$ 23. $\frac{1}{2}$ 24. $\frac{-5\sqrt{2}}{4}$ 25. $\frac{\sqrt{2}}{2}$

B. 1. $-i$ 2. 1 3. i 4. -1 5. 1 6. $-i$ 7. 1 8. i 9. $-i$.

C. 1. $k = 4n$ 2. $k = 4n + 1$ 3. $k = 4n + 2$ 4. $k = 4n + 3$

ASSIGNMENT #3

1. a. $3 + 2i$, $a = 3$, $b = 2$; b. $-5 + 6i$, $a = -5$, $b = 6$;
c. $17 + 0i$, $a = 17$, $b = 0$; d. $0 + (-3)i$, $a = 0$, $b = -3$
e. $5 + (-1)i$, $a = 5$, $b = -1$; f. $2 + i$, $a = 2$, $b = 1$
g. $8 + \sqrt{5}\,i$, $a = 8$, $b = \sqrt{5}$; h. $15 + (-20)i$, $a = 15$, $b = -20$
i. $8 + (-\sqrt{5})i$, $a = 8$, $b = -\sqrt{5}$ j. $1 + (-1)i$, $a = 1$, $b = -1$

2. When $b = 0$; 3. When $a = 0$.

ASSIGNMENT #4

A. $x = 7$, $y = 3$ B. $x = -2$, $y = 5$ C. $x = 8$, $y = -4$ D. $x = -5$, $y = 0$
E. $x = 0$, $y = 19$ F. $x = -2$, $y = 5$ G. $x = 2$, $y = 7$

ASSIGNMENT # 5

1. a. $2 + 10i$ b. $2 + 8i$ c. $-9 + 2i$ d. $9 - 2i$
2. a. $15 + 3i$ b. $15 + 5i$ c. $4 - 15i$ d. $-4 + 15i$
3. a. $1 + 2i$ b. $-1 - 4i$ c. $3 - i$ d. $-.3 - .1i$
4. a. $1 + i$ b. $-1 + 3i$ c. $2 + 2i$ d. $-1 + i$
5. a. $12 - 2i$ b. $2 + 2i$ c. $35 - 14i$ d. $\frac{35}{29} + \frac{14}{29}i$
6. a. $-7 + 7i$ b. $-5 - 7i$ c. $6 - 42i$ d. $\frac{3}{25} + \frac{21}{25}i$
7. a. $12i$ b. 10 c. -61 d. $\frac{11}{61} - \frac{60}{61}i$

a. $14i$ b. -2 c. -50 d. $\frac{24}{25} + \frac{7}{25}i$

9. a. $1 + 10i$ b. $-5 + 2i$ c. $-30 + 10i$ d. $\frac{18}{25} + \frac{26}{25}i$

10. a. $6 + 8i$ b. $-6i$ c. $2 + 24i$ d. $\frac{8}{29} - \frac{9}{29}i$

11. a. $-1 + 11i$ b. $3 - i$ c. $-32 - 4i$ d. $\frac{7}{10} - \frac{2}{5}i$

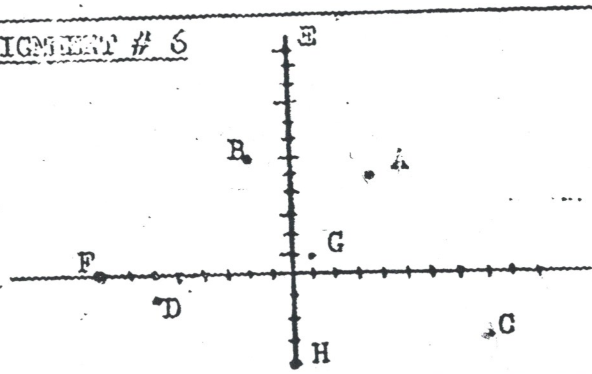
12. a. $-3 + 6i$ b. $-3 + 8i$ c. $7 + 3i$ d. $-7 - 3i$

13. $6 + 12i$; 14. $3 - 8i$; 15. $-3 + 5i$; 16. -8; 17. $4 - 3i$; 18. $-2 - 6i$

19. $4 - i\sqrt{3}$; 20. 3; 21. $\sqrt{5} + i$; 22. $8 + 4i$; 23. $4i$; 24. $\sqrt{7}$

25. $a - bi$; 26. $x + yi$; 27. the conjugate

ASSIGNMENT # 6



2. imaginary
3. real
4. imaginary
5. real

ASSIGNMENT # 7

1. $\sqrt{29}$; 2. $3\sqrt{10}$
3. 10 4. 17
5. 5 6. $2\sqrt{2}$
7. 0 8. $\sqrt{145}$
9. $\sqrt{26}$

ASSIGNMENT #8

1. $16 + 30i$; 2. $18 + 26i$; 3. $2 - 11i$; 4. 625;
5. 81; 6. $-644 + 960i$

ASSIGNMENT # 9

1. a. Two real roots; b. Two real roots; c. Two real roots
d. Two complex roots; e. Two complex roots; f. One real root;
g. Two real roots; h. One real root; i. Two complex roots;
j. Two real roots.

2. a. $\{6, 3\}$ b. $\left\{-\frac{1}{2} + \frac{\sqrt{3}}{2}i, -\frac{1}{2} - \frac{\sqrt{3}}{2}i\right\}$ c. $\left\{-\frac{1}{2} + \frac{\sqrt{11}}{2}i, -\frac{1}{2} - \frac{\sqrt{11}}{2}i\right\}$

d. $\left\{-\frac{5}{2} + \frac{\sqrt{37}}{2}, -\frac{5}{2} - \frac{\sqrt{37}}{2}\right\}$; e. $\left\{\frac{5}{4} + \frac{\sqrt{31}}{4}i, \frac{5}{4} - \frac{\sqrt{31}}{4}i\right\}$

f. $\left\{-\frac{3}{2} + \frac{\sqrt{285}}{10}, -\frac{3}{2} - \frac{\sqrt{285}}{10}\right\}$; g. $\{4, -1\}$

ALGEBRA 2

COMPLEX NUMBERS--TRIAL RUN

I. Define: 1. The imaginary unit
2. A pure imaginary number
3. A complex number
4. The conjugate of a complex number
5. The absolute value of a complex number

II. Evaluate

1. $\sqrt{-81}$
2. $2\sqrt{-18}$
3. $\sqrt{-5} + \sqrt{-3} + \sqrt{+125}$
4. $\sqrt{-4}\sqrt{-16}$
5. $(\sqrt{-3})^2$
6. $\sqrt{-4x^4} - \sqrt{-36x^4} + \sqrt{-100x^4}$
7. $5\sqrt{-75}$
8. $\dfrac{\sqrt{-18}}{\sqrt{-2}}$
9. $2i - 6i$
10. $(3i)(5i)$
11. $(2i)(3i)(4i)$
12. $(5i)^2$
13. $\dfrac{7}{i}$
14. $(-3i)(-2i)$
15. $\dfrac{2\sqrt{-2}}{\sqrt{-8}}$
16. $i + 2i^2 + 3i^3 + 4i^4$
17. i^3
18. i^{42}
19. i^{88}

III. For what values of k, such that $k > 0$, does:

(a) $i^k = -i$ (b) $i^k = 1$ (c) $i^k = i$ (d) $i^k = -1$

IV Simplify each of the following:

1. $(3 + 8i) + (8 - 2i)$
2. $(6 - 2i) - (3 + 7i)$
3. $(5 - 6i) - (2 - 8i)$
4. $(2 + i)(6 - i)$
5. $(6 + i) \div (2 + i)$
6. $(4 + i)^2$
7. $(1 + i)^3$
8. $(7 - 3i)(2 - 5i)$
9. $\dfrac{3 + 2i}{1 - i}$
10. $(5 + 2i) \div (6 - 3i)$
11. $\overline{3 + 2i}$
12. $\overline{5 - 6i}$
13. $\overline{18}$
14. $\overline{13i}$
15. $|3 + 4i|$
16. $|1 + i|$
17. $|-5 - 2i|$
18. $|-2i|$
19. $|-13|$
20. $(3 + 2i)^4$
21. $(1 + i)^2$
22. $(1 + i)^{-1}$

V. Solve for (x,y):

1. $(3x + 2yi) + (7 - 5i) = -5 + 3i$ 2. $(4x + 5yi) - (7x - 2i) = (8 - 18i) + 4$

3. $3x + 2 - 7yi + 1 = 2 - 4x + 8i$

VI. Graph these complex numbers on a complex coordinate system:

1. $3 + 2i$ 2. $6 - i$ 3. $-1 - 4i$ 4. $6i$ 5. -8

VII. Determine the nature of the roots of the solution of these quadratic equations:

1. $9x^2 + 6x + 1 = 0$ 2. $3x^2 - 2x - 4 = 0$ 3. $4x^2 - 5x = +6$

VIII. Solve each of the following quadratic equations:

1. $3x^2 - 2x + 5 = 0$ 2. $2x^2 - 7x + 9 = 0$ 3. $2x^2 + 3x = 2$

ANSWERS

I. 1. $\sqrt{-1}$ 2. A number of the form bi, where b is a real number not equal to zero and $i = \sqrt{-1}$

3. A number of the form a + bi where a and b are real numbers and $i = \sqrt{-1}$.

4. For $a \pm bi$, the conjugate is $a \mp bi$.

5. For $a \pm bi$, the absolute value is $\sqrt{a^2 + b^2}$

II. 1. $9i$ 2. $6i\sqrt{2}$ 3. $i\sqrt{3} - 4i\sqrt{5}$ 4. -8 5. -3 6. $6ix^2$

7. $25i\sqrt{3}$ 8. 3 9. $-4i$ 10. -15 11. $-24i$ 12. -25 13. $-7i$

14. -6 15. 1 16. $2 - 2i$ 17. $-i$ 18. -1 19. 1

III. (a) $4n + 3$ (b) $4n$ (c) $4n + 1$ (d) $4n + 2$

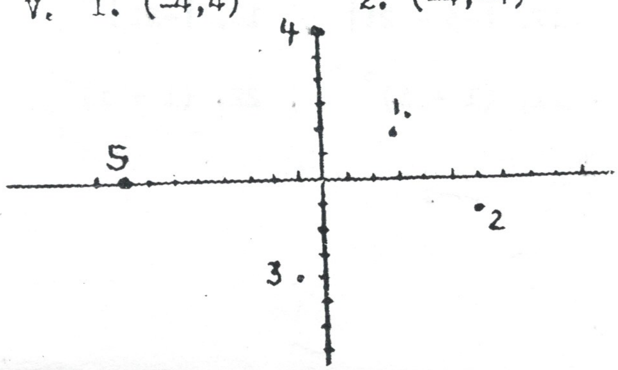
IV. 1. $11 + 6i$ 2. $3 - 9i$ 3. $3 + 2i$ 4. $13 + 4i$

5. $\frac{13 - 4i}{5}$ 6. $15 + 8i$ 7. $-2 + 2i$ 8. $-1 - 4i$ 9. $\frac{1 + 5i}{2}$

10. $\frac{8 + 9i}{15}$ 11. $3 - 2i$ 12. $5 + 6i$ 13. 18 14. $-13i$ 15. 5

16. $\sqrt{2}$ 17. $\sqrt{29}$ 18. 2 19. 13 20. $-119 + 120i$ 21. $2i$ 22. $\frac{1 - i}{2}$

V. 1. $(-4,4)$ 2. $(-4,-4)$ 3. $(-\frac{1}{7}, -\frac{8}{7})$



VII. 1. 2 equal real rts 2. Two real roots 3. Two complex roots

VIII. 1. $\frac{1 \pm i\sqrt{14}}{3}$

2. $\frac{7 \pm i\sqrt{23}}{4}$ 3. $\{-2, \frac{1}{2}\}$