FACTORING

OBJECTIVES

I. Given a polynomial, find the common monomial or binomial factor

II. Given a binomial of the form $a^n \pm b^n$, factor the binomial when possible

III. Factor a given perfect trinomial square

IV. Factor a trinomial which is the product of two binomials

V. Group factorable polynomials of more than three terms so that they fit one of the factor forms and then factor the polynomial

VI. Use the factor theorm to determine whether or not a polynomial has a binomial of the form $x - a$ as a factor.

VII. Use the process of factoring to solve equations where the factors are obvious

VIII. Simplify rational expressions by using factoring.

SECTION 1 THE COMMON FACTOR - A MONOMIAL

This section deals with the distributive principle in reverse. If a polynomial consists of terms each of which has a common factor, that common factor may be taken out. Then the product can be written as a monomial times a binomial. Study the following examples:

1. $7x^2 + 14x = 7x(x + 2)$
2. $35a^3 - 25a^2b^2 = 5a^2(7a - 5b^2)$
3. $6x^3y + 24xy^2 - 36x^4y^5 = 6xy(x^2 + 4y - 6x^3y^4)$

EXERCISE 1 Each of the following polynomials has a largest common monomial factor. Express each as a product of a monomial times a binomial.

1. $3x^2 - 6xy$
2. $3ab - 6ab + 9b$
3. $a^3 + 3a^2b + ab^2$
4. $7x - 7$
5. $3a^2 + 3a$
6. $2x^3 + 4$
7. $a^2 + ab - ax$
8. $3a^2b + 3ab^2 + 3b^4$
9. $3x^3 - 15x^2$
10. $24a - 16b + 24c$
11. $4x^3 - 8ax^2 - 12a^2x^2$
12. $a^2y - 4aby + b^2y$
13. $3x^2yz^5 + 10xyz^3$
14. $12x^2y - 24xy^2$
15. $a^2b^2c^2 + abc$
16. $4a^2b^2 + 6a^2b - 2a^6$
17. $18 - 36a$
18. $a^2b^2c^2 + a^3b^3c^3 - a^4b^4c^4$

SECTION 2 THE COMMON FACTOR--A BINOMIAL

With certain sums or differences of an even number of terms it is possible to group the terms, factor out a common monomial, and be left with a common binomial factor. Study the following examples carefully:

Example 1: $cx + cy + bx + by$

From the first two terms, factor out a c; and from the second two terms factor out a b.

$c(x + y) + b(x + y)$

Now we have a binomial. The common factor the common factor: $(x + y)$

$(x + y)(c + b)$

Check the result. Multiply $(x + y)(c + b)$
Notice: The product is: $cx + cy + bx + by$

Example 2: $ax - ay - bx + by$

$a(x - y) - b(x - y)$ WATCH THOSE SIGNS!

$(x - y)(a - b)$

Example 3: $6x + 2a - 3x^2 - ax$

$6x - 3x^2 + 2a - ax$ Change order of the terms

$3x(2 - x) + a(2 - x)$

$(2 - x)(3x + a)$.

EXERCISE 2 Each of the following can be factored as the product of two binomials. There is no set method that can be stated. It is a matter of creativity. Do what you can. Check your answers by multiplication.

1. $ax - bx + ay - by$
2. $2ax + 6a + x + 3$
3. $a^2 + ab - 2a - 2b$
4. $5x + 10y - 6ax - 12ay$
5. $b + ab - 5 - 5a$
6. $xz - yz + 2x - 2y$
7. $4y^2 - 4yz + z^3y - z^4$
8. $h^3 + 4h^2 - 9h - 36$
9. $cx^2 - cy^2 + 4y^2 - 4x^2$
10. $3a^2 - 2ax - 3a + 2x$
11. $2x^3 + x^2 - 6x - 3$
12. $3a^2 - b^2 + 3ab - a^2b$

SECTION 3 — FACTORING THE DIFFERENCE OF TWO PERFECT SQUARES

Notice, when $(a + b)(a - b)$ is expanded, the result is $a^2 + ab - ab - b^2$. The two middle terms, ab and $-ab$, fall out, leaving $a^2 - b^2$.

When factoring, we think the reverse of multiplication. We look for polynomials which when multiplied give the result at hand.

Study the following examples. Check out the resulting products to see if the factorization is correct.

Examples:

1. $4 - a^2 = (2 + a)(2 - a)$
2. $x^2 - 36 = (x + 6)(x - 6)$
3. $100x^2 - 25 = 25(4x^2 - 1) = 25(2x + 1)(2x - 1)$
4. $49x^2 - 81y^2 = (7x + 9y)(7x - 9y)$
5. $x^4 - 1 = (x^2 + 1)(x^2 - 1)$
   $= (x^2 + 1)(x + 1)(x - 1)$
6. $x^2 - 5 = (x + \sqrt{5})(x - \sqrt{5})$ REALLY, this could go on forever. We will limit our work to the set of rational numbers. No square root signs.
7. $a^2 + 2ab + b^2 - d^2 = (a^2 + 2ab + b^2) - d^2$
   $= (a + b)^2 - d^2$
   $= (a + b + d)(a + b - d)$
8. $18x^2 - 8y^2 = 2(9x^2 - 4y^2)$
   $= 2(3x + 2y)(3x - 2y)$

In general:
1. Factor out any common factor.
2. Study the remaining polynomial to see if it contains the <u>difference</u> of two perfect squares. If so, factor the result as the product of the sum and difference of the two square roots.

EXERCISE 3 Factor each of the following:

1. $x^2 - 49$
2. $9b^2 - 4$
3. $25 - d^2$
4. $y^2 - 81$
5. $(x + y)^2 - z^2$
6. $25 - (x - y)^2$
7. $16x^2y^2 - 9z^2$
8. $y^2 + 2y + 1 - 81$
9. $4w^2 - 36z^2$
10. $x^4 - 16$
11. $27x^2 - 108$
12. $c^2 - a^2 + 2ab - b^2$
13. $x^{16} - 1$
14. $1 - a^8$
15. $3a^2 - 27 - 6ab + 3b^2$

This section is generally a review. There is no set method that can be applied in order to factor a trinomial. Some trinomials can be factored. Some cannot. ALWAYS look for a common factor. From that point on, it is a matter of being creative. A check on the work is multiplication to determine whether or not the factors do, indeed, give the original trinomial. Study the following examples.

Examples: 1. $x^2 + 7x + 10 = (x + 5)(x + 2)$

2. $6x^2 - 7x - 3 = (3x + 1)(2x - 3)$

3. $4x^2 + 20x + 25 = (2x + 5)(2x + 5) = (2x + 5)^2$

4. $4x^2 - 36x + 81 = (2x - 9)(2x - 9) = (2x - 9)^2$

5. $9x^2 - 48x + 64 = (3x - 8)(3x - 8) = (3x - 8)^2$

6. $2a^2 - 4ab + 2b^2 = 2(a^2 - 2ab + b^2) = 2(a - b)^2$

EXERCISE 4 Give the prime factorization for each of the following:

1. $4a^2 + 16a + 16$	2. $5a^2 - 30a + 45$	3. $y^2 + 4y - 12$
4. $18y^2 - 21y - 9$	5. $3x^2 + 5x + 2$	6. $x^2 - x - 12$
7. $x^2 + 6x + 9$	8. $x^2 - 10x + 25$	9. $c^2 - 8c + 16$
10. $y^2 + 16y + 64$	11. $u^2v^2 + 3uv + 9v^2$	12. $r^2x^2 - 4rx + 4$
13. $9s^2 - 24st + 16t^2$	14. $c^2 - 13c + 12$	15. $5p^2 + 13p - 18$

SECTION 5 FACTORING A BINOMIAL OF THE FORM $a^3 + b^3$ OR $a^3 - b^3$

THEOREM:

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

LOOK FOR A PATTERN
MEMORIZE

PLEASE memorize the above theorem. To prove the theorem, simply multiply the two factors. This is a most important piece of information. Here, for the first time in this L.A.P., a new concept is being introduced.

DO NOT TURN THE PAGE UNTIL YOU HAVE MEMORIZED THE ABOVE THEOREM.

Study the following exampl . In each case the theorem fo this section is being applied.

Examples: 1. $x^3 + 8 = x^3 + 2^3 = (x + 2)(x^2 - 2x + 4)$

2. $8x^3 - 27y^3 = (2x)^3 - (3y)^3 = (2x - 3y)(4x^2 + 6xy + 9y^2)$

3. $27x^3 - y^3 = (3x - y)(9x^2 + 3xy + y^2)$

4. $4x^3 + 32y^3 = 4(x^3 + 8y^3) = 4(x^3 + (2y)^3)$
$= 4(x + 2y)(x^2 - 2xy + 4y^2)$

EXERCISE 5 Give the prime factorization for each of the following: (Do these fast. With the theorem memorized there should be no trouble.)

1. $s^3 + 27$ 2. $r^3 - 8$ 3. $64x^3 - 1$ 4. $y^3 - 125$

5. $8w^3 - 27x^3$ 6. $1000w^3 + 8x^3$ 7. $(x + y)^3 - 8$ 8. $y^3 - 64w^3$

9. $8t^3 - 729$ 10. $27x^3 + (y + 5)^3$ 11. $1 - (x + y)^3$

SECTION 6 FACTORING BINOMIALS OF THE FORM $a^n \pm b^n$ WHEN n IS ODD

THEOREMS

$$a^5 + b^5 = (a + b)(a^4 - a^3b + a^2b^2 - ab^3 + b^4)$$

$$a^5 - b^5 = (a - b)(a^4 + a^3b + a^2b^2 + ab^3 + b^4)$$

$$a^7 + b^7 = (a + b)(a^6 - a^5b + a^4b^2 - a^3b^3 + a^2b^4 - ab^5 + b^6)$$

$$a^7 - b^7 = (a - b)(a^6 + a^5b + a^4b^2 + a^3b^3 + a^2b^4 + ab^5 + b^6)$$

The above pattern holds for all odd powers. Notice some particulars about the pattern:

1. One factor is a binomial--just like the original without the powers.
2. The second factor has the first term with descending exponents. . . first term; ascending exponents. . . second term.
3. If the binomial has a + sign, the signs in the second factor alternate.
4. If the binomial has a - sign, The signs in the second factor are all +.

The above pattern holds for all binomials of the form: $a^n \pm b^n$ where n is ODD. The proof lies in multiplication. Preform the multiplication and see for yourself that it works!

EXERCISE 6 Factor each of the following. Some sums and differences of cubes are thrown in. . . after all, 3 is odd!

1. $x^5 - 32$
2. $x^9 + y^9$
3. $x^7 + 128$
4. $x^5 + 1$
5. $x^7 - y^7$
6. $x^9 - 1$
7. $x^7 - 128$
8. $x^3 + 8$
9. $8x^3 + y^3$
10. $x^5 + y^5$
11. $243 - x^5$
12. $a^7 - b^7$
13. $64y^3 - 27$
14. $x^5 - 1$
15. $x^7 - 1$

SECTION 7 FACTORING BINOMIALS OF THE FORM $a^n \pm b^n$ WHEN n IS EVEN

CASE 1: For the sign of the binomial +

Study the following examples. Notice that the trick is to make the power of each term of the binomial <u>ODD</u>. If this is not possible, the binomial usually cannot be factored. There are a few freaky cases but those will not be considered here.

Examples: 1. $x^6 + y^6 = (x^2)^3 + (y^2)^3$

$= (x^2 + y^2)(x^4 - y^2y^2 + y^4)$

2. $x^2 + y^2 =$ . . . No dice. . . cannot be factored.

3. $x^4 + y^4 =$ Prime. . . cannot be factored

4. $x^{10} + y^{10} = (x^2)^5 + (y^2)^5$

$= (x^2 + y^2)((x^2)^4 - (x^2)^3(y^2) + (x^2)^2(y^2)^2 - (x^2)(y^2)^3 + (y^2)^4)$

$= (x^2 + y^2)(x^8 - x^6y^2 + x^4y^4 - x^2y^6 + y^8)$

5. $x^8 + y^8 =$ . . . No dice, . . 8 cannot be written as a product of an odd number times an even number.

This binomial is generally prime

To find the other factors of the given polynomial, divide the polynomial by $(x + 4)$.

$$
\begin{array}{r}
x^2 + 6x + 5 \\
x + 4 \overline{\big) x^3 + 10x^2 + 29x + 20} \\
\underline{x^3 + 4x^2} \\
6x^2 + 29x \\
\underline{6x^2 + 24x} \\
5x + 20 \\
\underline{5x + 20} \\
0
\end{array}
$$

$x^2 + 6x + 5$ can be factored to: $(x + 5)(x + 1)$

Hence, the factors of $x^3 + 10x^2 + 29x + 20$ are $(x + 4)(x + 5)(x + 1)$

Check the result by multiplication.

Example 2 Factor the polynomial $x^3 - 9x^2 + 24x - 20$

(1) For what value of x is the value of the polynomial equal to zero?

For your information: Check only those numbers which divide the constant... In this case that number is 20.

(2) Notice: for $x = 2$: $8 - 36 + 48 - 20 = 0$

(3) Hence: $(x - 2)$ is a factor of the given polynomial.

(4) Divide to find the other factors:

$$
\begin{array}{r}
x^2 - 7x + 10 \\
x - 2 \overline{\big) x^3 - 9x^2 + 24x - 20} \\
\underline{x^3 - 2x^2} \\
-7x^2 + 24x \\
\underline{-7x^2 + 14x} \\
10x - 20 \\
\underline{10x - 20}
\end{array}
$$

(5) Factor $x^2 - 7x + 10$. $x^2 - 7x + 10 = (x - 5)(x - 2)$

Solution: $x^3 - 9x^2 + 24x - 20 = (x - 2)^2(x - 5)$

HAPPY CLUE: Any number that you pick to check out to find whether or not it makes the value of the polynomial zero, must be a factor of the constant term.

CASE 2: For the [illegible]

Study the following examples. The trick for this case is to first factor according to the rule for the difference of two squares. After that do what you need to do to make sure the factorization is prime.

Examples:

1. $x^4 - y^4 = (x^2 + y^2)(x^2 - y^2)$
$= (x^2 + y^2)(x + y)(x - y)$

2. $x^6 - y^6 = (x^3 + y^3)(x^3 - y^3)$
$= (x + y)(x^2 - xy + y^2)(x - y)(x^2 + xy + y^2)$

3. $x^8 - y^8 = (x^4 + y^4)(x^4 - y^4)$
$= (x^4 + y^4)(x^2 + y^2)(x^2 - y^2)$
$= (x^4 + y^4)(x^2 + y^2)(x + y)(x - y)$

4. $x^{10} - y^{10} = (x^5 + y^5)(x^5 - y^5)$
$= (x + y)(x^4 - x^3y + x^2y^2 - xy^3 + y^4)(x - y)$
$(x^4 + x^3y + x^2y^2 + xy^3 + y^4)$

EXERCISE 7 Give the prime factorization for each of the following:

1. $x^4 - 1$	2. $n^4 - 16$	2. $1 + 125x^6$	3. $y^4 + 16$
4. $x^6 - 27$	5. $x^4 - 4x^2$	6. $1 - 8x^6$	7. $x^2 + y^2$
8. $x^{10} - 1$	9. $x^{10} + 1$	10. $x^6 + 64$	11. $x^5 - 32$
12. $x^7 - y^7$	13. $x^9 + y^9$	14. $x^9 - 1$	15. $x^7 + 128$
16. $x^7 - 128$	17. $x^5 + 1$	18. $x^6 - 64$	19. $x^{12} - y^{12}$

SECTION 8 THE FACTOR THEOREM

The factor theorem: The binomial $(x - a)$ is a factor of a polynomial with lead coefficient 1, if and only if the value of the polynomial is zero for $x = a$.

Consider the polynomial: $x^3 + 10x^2 + 29x + 20$. Notice for $x = -4$;

$$(-4)^3 + 10(-4)^2 + 29(-4) + 20 = -64 + 160 - 116 + 20 = 0$$

By the factor theorem, $x + 4$ is a factor of the polynomial.

EXERCISE 8 Factor each of the following:

1. $x^3 - 10x^2 + 27x - 18$
2. $x^3 - 8x^2 + 21x - 18$
3. $x^3 - 27x + 10$ $x^3 - 0x^2 - 27x + 10$
4. $x^3 - 12x^2 + 29x - 18$
5. $x^3 - 6x^2 + 11x - 6$
6. $x^4 - 10x^3 + 35x^2 - 50x + 24$

SECTION 9 **USING FACTORING TO SOLVE QUADRATIC EQUATIONS**

This section is a bit of a review of Algebra 1. Study the examples and proceed to the exercises.

Example 1: Solve for x: $x^2 - 5x - 14 = 0$

Solution: (1) Factor: $x^2 - 5x - 14 = (x - 7)(x + 2) = 0$

(2) Set each factor equal to 0:

$x - 7 = 0$ or $x + 2 = 0$

(3) Solve for x: $x = 7$ or $x = -2$

Example 2: Solve for x: $2x^2 - 32 = 0$

Solution: $2(x^2 - 16) = 0$

$2(x + 4)(x - 4) = 0$

$x = -4$ or $x = 4$

EXERCISE 9 Solve each of the following equations for the given variable:

1. $x^2 + x - 6 = 0$
2. $x^2 - x - 42 = 0$
3. $r^2 - 15r + 54 = 0$
4. $x^2 - 8x = 20$
5. $a^2 - 5a = 14$
6. $x^2 + x - 90 = 0$
7. $3x^2 = 13x - 4$
8. $4y^2 = 36$
9. $3x^2 - 10x = -8$
10. $8x^2 + 16x + 6 = 0$
11. $x^2 = 25$
12. $(x + 2)^2 = 16$

SECTION 10 **SIMPLIFYING RATIONAL EXPRESSIONS**

Rational expressions are really fractions. Fractions can be simplified by dividing both numerator and denominator by the same value. Again, this section is a bit of a review. Study the examples. Check with your teacher if there is any problem.

SIMPLIFYING RATIONAL EXPRESSIONS

(1) Factor both numerator and denominator.

(2) Divide out like factors

Example. Simplify:

$$\frac{4x^3 + 24x^2 + 36x}{2x^4 - 18x^2} = \frac{4x(x^2 + 6x + 9)}{2x^2(x^2 - 9)}$$

$$= \frac{4x(x + 3)(x + 3)}{2x^2(x + 3)(x - 3)} = \frac{2(x + 3)}{x(x - 3)}$$

MULTIPLYING RATIONAL EXPRESSIONS

(1) Factor each numerator and denominator

(2) Divide out like factors

(3) Combine to form a single rational expression

Example: Simplify:

$$\frac{x^2 - 2x - 3}{x^2 - 3x} \cdot \frac{x^2 + 2x}{x^2 - 4} = \frac{(x - 3)(x + 1)}{x(x - 3)} \cdot \frac{x(x + 2)}{(x + 2)(x - 2)}$$

$$= \frac{x + 1}{x + 2}$$

DIVIDING RATIONAL EXPRESSIONS

(1) Invert the divisor; i.e. the second fraction

(2) Proceed as with multiplication of two rational expressions

Example: Simplify:

$$\frac{x^2 - 4}{x^2 + 2x} \div \frac{x^2 + x - 6}{2x + 4} = \frac{x^2 - 4}{x^2 + 2x} \cdot \frac{2x + 4}{x^2 + x - 6}$$

$$= \frac{(x + 2)(x - 2)}{x(x + 2)} \cdot \frac{2(x + 2)}{(x + 3)(x - 2)}$$

$$= \frac{2(x + 2)}{x(x + 3)} = \frac{2x + 4}{x^2 + 3x}$$

EXERCISE 10-A SIMPLIFY EACH RATIONAL EXPRESSION

1. $\dfrac{a^2 + ab}{a(a + b) - 2(a + b)}$

2. $\dfrac{x^2 - y^2}{x^2 + 2xy + y^2}$

3. $\dfrac{2x^2 - 8}{x^3 - 8}$

4. $\dfrac{3x^3 + 3y^3}{2x^3 - 2x^2y + 2xy^2}$

5. $\dfrac{6y + 18}{y^2 + 6y + 9}$

6. $\dfrac{x^3 - 1}{x^2 + x + 1}$

EXERCISE 10-B **MULTIPLYING RATIONAL EXPRESSIONS**

1. $\dfrac{5x - 5}{3} \cdot \dfrac{9}{x - 1}$

2. $\dfrac{x + 2}{x^2 - 4} \cdot \dfrac{x^2 + 4x + 4}{3x + 4}$

3. $\dfrac{y^2 - 36}{7 - y} \cdot \dfrac{y - 7}{6 - y}$

4. $\dfrac{x^2 + 7x + 12}{x^2 + 2x - 8} \cdot \dfrac{x^2 + 3x - 10}{x^2 + 8x + 15}$

5. $\dfrac{x^3 + 8}{x + 2} \cdot \dfrac{x^2 - 5x + 1}{x^2 - 2x + 4}$

6. $\dfrac{x^2 - 16}{2x + 8} \cdot \left(\dfrac{x^2 + 8x + 16}{x + 4}\right)^{-1}$

EXERCISE 10-C **DIVISION OF RATIONAL EXPRESSIONS**

1. $\dfrac{x^2 - 2x - 15}{x^2 - 9} \div \dfrac{2x - 10}{x + 3}$

2. $\dfrac{x^2 - 3x}{9 - x^2} \div \dfrac{x}{3 + x}$

3. $\dfrac{x^2 + 2x}{4x - 5} \div \dfrac{2x^2 + 4x}{16x - 20}$

4. $\dfrac{x^2 - x - 6}{x^2 - 4} \div \dfrac{x - 3}{x - 2}$

5. $\dfrac{x^3 + y^3}{x^3 - y^3} \div \dfrac{x^2 + 2xy + y^2}{x^2 - y^2}$

6. $\dfrac{x^2 - y^2}{y^2 - z^2} \div \dfrac{az + xz}{ax - az}$

7. $\dfrac{3y^2 + 14y - 5}{4y^2 - 9} \div \dfrac{3y - 1}{2y + 3}$

8. $\dfrac{x + 2}{2x - 3} \div \dfrac{x^2 - 4}{2x^2 - 3x}$

This is the end of this L.A.P. Take the Trial Run. If you are not sure about any concepts consult with your teacher. Then take the test on this topic.

L.A.P. ANSWERS

EXERCISE 1

1. $3x(x - 2y)$ 2. $3b(3 - a)$ 3. $a(a^2 + 3ab + b^2)$ 4. $7(x - 1)$
5. $3a(a + 1)$ 6. $2(x^7 + 2)$ 7. $a(a + b - x)$
8. $3b(a^2 + ab + b^3)$ 9. $3x(x - 5)$ 10. $8(3a - 2b + 3c)$
11. $4x^2(x - 2a - 3a^2)$ 12. $y(a^2 - 4ab + b^2)$ 13. $3xyz^3(xz^2 + 6)$
14. $12xy(x - 7y)$ 15. $ab(bc + 1)$ 16. $2a^2(2b^2 + 3b - a^4)$
17. $(1 - 2a)$ 18. $a^2b^2c^2(1 + abc - a^2b^2c^2)$

EXERCISE 2

1. $(a - b)(x + y)$ 2. $(x + 3)(2a + 1)$ 3. $(a - 2)(a + b)$
4. $(5 - 6a)(x + 2y)$ 5. $(a + 1)(b - 5)$ 6. $(z + 2)(x - y)$
7. $(4v + z^3)(y - z)$ 8. $(h + 4)(h + 3)(h - 3)$
9. $(a - 4)(x + y)(x - y)$ 10. $(3a - 2x)(a - 1)$ 11. $(x^2 - 3)(2x + 1)$
12. This polynomial has no factors. It is prime.

EXERCISE 3

1. $(x + 7)(x - 7)$ 2. $(3b + 2)(3b - 2)$ 3. $(5 + d)(5 - d)$
4. $(y + 9)(y - 9)$ 5. $(x + y + z)(x + y - z)$
6. $(5 + x - y)(5 - x + y)$ 7. $(4xy + 3z)(4xy - 3z)$ 8. $(y + 10)(y - 10)$
9. $4(w + 3z)(w - 3z)$ 10. $(x^2 + 4)(x + 2)(x - 2)$ 11. $27(x + 2)(x - 2)$
12. $(c + a - b)(c - a + b)$ 13. $(x^4 + 1)(x^2 + 1)(x + 1)(x - 1)(x^8 + 1)$
14. $(1 + a^4)(1 + a^2)(1 + a)(1 - a)$ 15. $3(a - b + 3)(a - b - 3)$

EXERCISE 4

1. $4(a + 2)^2$ 2. $5(a - 3)^2$ 3. $(y + 6)(y - 2)$
4. $3(2y - 3)(3y + 1)$ 5. $(3x + 2)(x + 1)$ 6. $(x - 4)(x + 3)$
7. $(x + 3)^2$ 8. $(x - 5)^2$ 9. $(a - 4)^2$ 10. $(y + 8)^2$
11. $v(u^2v + 3u + 9v)$ 12. $(rx - 2)^2$ 13. $(3s - 4t)^2$
14. $(c - 12)(c - 1)$ 15. $(p + 18)(p - 1)$

EXERCISE 5

1. $(s + 3)(s^2 - 3s + 9)$
2. $(r - 2)(r^2 + 2r + 4)$
3. $(4x - 1)(16x^2 + 4x + 1)$
4. $(y - 5)(y^2 + 5y + 25)$
5. $(2w - 3x)(4w^2 + 6wx + 9x^2)$
6. $8(5w + x)(25w^2 - 5wx + x^2)$
7. $(x + y - 2)((x + y)^2 + 2(x + y) + 4)$
8. $(y - 4w)(y^2 + 4wy + 16w^2)$
9. $(2t - 9)(4t^2 + 18t + 81)$
10. $(3x + y + 5)(9x^2 - 3x(y + 5) + (y + 5)^2)$
11. $(1 - x - y)(1 + x + y + (x + y)^2)$

EXERCISE 6

1. $(x - 2)(x^4 + 2x^3 + 4x^2 + 8x + 16)$
2. $(x + y)(x^2 - xy + y^2)(x^6 - x^3y^3 + y^6)$
3. $(x + 2)(x^6 - 2x^5 + 4x^4 - 8x^3 + 16x^2 - 32x + 64)$
4. $(x + 1)(x^4 - x^3 + x^2 - x + 1)$
5. $(x - y)(x^6 + x^5y + x^4y^2 + x^3y^3 + x^2y^4 + xy^5 + y^6)$
6. $(x - 1)(x^2 + x + 1)(x^6 + x^3 + 1)$
7. $(x - 2)(x^6 + 2x^5 + 4x^4 + 8x^3 + 16x^2 + 32x + 64)$
8. $(x + 2)(x^2 - 2x + 4)$
9. $(2x + y)(4x^2 - 2xy + y^2)$
10. $(x + y)(x^4 - x^3y + x^2y^2 - xy^3 + y^4)$
11. $(3 - x)(81 + 27x + 9x^2 + 3x^3 + x^4)$
12. $(a - b)(a^6 + a^5b + a^4b^2 + a^3b^3 + a^2b^4 + ab^5 + b^6)$
13. $(4y - 3)(16y^2 + 12y + 9)$
14. $(x - 1)(x^4 + x^3 + x^2 + x + 1)$
15. $(x - 1)(x^6 + x^5 + x^4 + x^3 + x^2 + x + 1)$

EXERCISE 7

1. $(x^2 + 1)(x + 1)(x - 1)$
2. $(x^2 + 4)(n + 2)(n - 2)$
2. $(1 + 5x^2)(1 - 5x^2 + 25x^4)$
3. Prime
4. $(x^2 - 3)(x^4 + 3x^2 + 9)$
5. $x^2 (x + 2)(x - 2)$
6. $(1 - 2x^2)(1 + 2x^2 + 4x^4)$
7. Prime
8. $(x + 1)(x^4 - x^3 + x^2 - x + 1)(x - 1)(x^4 + x^3 + x^2 + x + 1)$
9. $(x^2 + 1)(x^8 - x^6 + x^4 - x^2 + 1)$
10. $(x^2 + 4)(x^4 - 4x^2 + 16)$
11. $(x - 2)(x^4 + 2x^3 + 4x^2 + 8x + 16)$

12. $(x - y)(x^6 + x^5y + x^4y^2 + x^3y^3 + x^2y^4 + xy^5 + y^6)$
13. $(x + y)(x^2 - xy + y^2)(x^6 - x^3y^3 + y^6)$
14. $(x - 1)(x^2 + x + 1)(x^6 + x^3 + 1)$
15. $(x + 2)(x^6 - 2x^5 + 4x^4 + 8x^3 + 16x^2 - 32x + 64)$
16. $(x - 2)(x^6 + 2x^5 + 4x^4 + 8x^3 + 16x^2 + 32x + 64)$
17. $(x + 1)(x^4 - x^3 + x^2 - x + 1)$
18. $(x - 2)(x^2 + 2x + 4)(x + 2)(x^2 - 2x + 4)$
19. $(x^2 + y^2)(x^4 - x^2y^2 + y^4)(x + y)(x^2 - xy + y^2)(x - y)(x^2 + xy + y^2)$

EXERCISE 8

1. $(x - 1)(x - 3)(x - 6)$
2. $(x - 2)(x - 3)^2$
3. $(x - 5)(x^2 + 5x - 2)$
4. $(x - 1)(x - 2)(x - 9)$
5. $(x - 1)(x - 2)(x - 3)$
6. $(x - 1)(x - 2)(x - 3)(x - 4)$

EXERCISE 9

1. $\{-3, 2\}$
2. $\{-6, 7\}$
3. $\{6, 9\}$
4. $\{-2, 10\}$
5. $\{-2, 7\}$
6. $\{-10, 9\}$
7. $\{1/3, 4\}$
8. $\{-3, 3\}$
9. $\{4/3, 2\}$
10. $\{-3/2, -1/2\}$
11. $\{-5, 5\}$
12. $\{-6, 2\}$

EXERCISE 10-A

1. $\frac{a}{a - 2}$
2. $\frac{x - y}{x + y}$
3. $\frac{2(x + 2)}{x^2 + 2x + 4}$
4. $\frac{3(x + y)}{2x}$
5. $\frac{6}{y + 3}$
6. $x - 1$

EXERCISE 10-B

1. 15
2. $\frac{(x + 2)^2}{(3x + 4)(x - 2)}$
3. $y + 6$
4. 1
5. $x^2 - 5x + 1$
6. $\frac{x - 4}{2(x + 4)}$

EXERCISE 10-C

1. $\frac{x + 3}{2(x - 3)}$
2. -1
3. 2
4. 1
5. $\frac{x^2 - xy + y^2}{x^2 + xy + y^2}$
6. $(x^2 - y^2)$[illegible]
7. $x + 5$ [illegible]
8. $\frac{x}{x - 2}$

ALGEBRA 2

FACTORING TRIAL RUN

I. Factor each of the following:

1. $24a^4b^3 - 36a^3b^2 + 12a^2b$
2. $3a^2(a - 2b) + 5a(a - 2b) + (a - 2b)$
3. $5x(a^2 + a - 3) + 2y(a^2 + a - 3) - 7(a^2 + a - 3)$
4. $64x^2y^2 - 25$
5. $8a^3 - 1$
6. $4a^2 + 25$
7. $x^8 - 1$
8. $125 + x^3$
9. $32x^5 - a^5$
10. $64a^6 + b^6$
11. $25x^2y^2 - 10xy + 1$
12. $ay - xy - a + x$
13. $a^2 - 2ab + b^2 - 4$
14. $2a^2 + a - 3$
15. $64a^2b^3c^4 - 25b^3c^4$
16. $x^5 + y^5$
17. $a^5 - 1$
18. $9 - (a + b)^2$
19. $9x^2 + 12xy + 4y^2$
20. $x^2(a + 2y) - (a + 2y)$
21. $a^2b^2 - a^2 - b^2 + 1$
22. $(a + b)a + (a + b)b$
23. $w^5 - w^4 - 4w + 4$
24. $64a^3 - 27$
25. $x^2 + 5x - 14$
26. $2w^2 - 7w + 3$
27. $6x^2 - 5x - 6$
28. $6x^4y^4 + 17x^2y^2 + 5$
29. $14a^2 - 16a + 5a - 15$
30. $15p^2 - p - 6$
31. $x^{10} - y^{10}$
32. $243 - 32x^5$

I. Use factoring to solve each of the following equations:

1. $x^2 - 3x - 10 = 0$
2. $6a^2 - a - 2 = 0$
3. $2x^2 - x - 3 = 0$
4. $5x^3 - 125x = 0$

III. Simplify each of the following:

1. $\dfrac{4x^3 + 10x^2 - 6x}{6x^2 - 4x}$
2. $\dfrac{7a^2b^2 - 7a^2bc}{c^2 - b^2}$
3. $\dfrac{a^3 - b^3}{3a + 6b} \cdot \dfrac{a^2 + 3ab + 2b^2}{a^2 - b^2}$
4. $\dfrac{x^4 - 1}{x^2 + 2x - 3} \cdot \dfrac{x^2 + 7x + 12}{5x^3 + 5x} \div \dfrac{10x}{2x + 2}$
5. $\dfrac{3a^2b - 3ab^2}{a^2 - b^2} \div \dfrac{9a^2b^2}{a^2b + ab^2}$
6. $\dfrac{x^2 + xy - 2y^2}{10x} \div \dfrac{2x^2 + 3xy - 2y^2}{x^3 + y^3} \cdot \dfrac{10x^2 - 35xy + 15y^2}{x^2 - y^2}$

IV. The Factor Theorem:

1. Which of the facotrs listed could not be a factor of $x^3 - 4x^2 - 19x - 14$?

a) $(x + 1)$ b) $(x + 2)$ c) $(x + 3)$ d) $x + 4)$ e) $(x - 7)$

2. Use the Factor Theorem to find the factors of: $x^4 - x^3 - 7x^2 + x + 6$

ANSWERS:

I. 1. $12a^2b(2ab - 1)(ab - 1)$ 2. $(a - 2b)(3a^2 + 5a + 1)$

3. $(a^2 + a - 3)(5x + 2y - 7)$ 4. $(8xy + 5)(8xy - 5)$

5. $(2a - 1)(4a^2 + 2a + 1)$ 6. prime

7. $(x^4 + 1)(x^2 + 1)(x + 1)(x - 1)$ 8. $(25 - 5x + x^2)(5 + x)$

9. $(2x - a)(16x^4 + 8ax^3 + 4a^2x^2 + 2a^3x + a^4)$

10. $(4a^2 + b^2)(16a^4 - 4a^2b^2 + b^4)$ 11. $(5xy - 1)(5xy - 1)$

12. $(y - 1)(a - x)$ 13. $(a - b + 2)(a - b - 2)$

14. $(2a + 3)(a - 1)$ 15. $64b^3c^4(a + 2)(a - 2)$

16. $(x + y)(x^4 - x^3y + x^2y^2 - xy^3 + y^4)$

17. $(a - 1)(a^2 + a + 1)(a + 1)(a^2 - a + 1)$

18. $(3 + a + b)(3 - a - b)$ 19. $(3x + 2y)^2$

20. $(a + 2y)(x + 1)(x - 1)$ 21. $(b + 1)(b - 1)(a + 1)(a - 1)$

22. $(a + b)^2$ 23. $(w^2 + 2)(w^2 - 2)(w - 1)$

24. $(4a - 3)(16a^2 + 12a + 9)$ 25. $(x + 7)(x - 2)$

26. $(2w - 1)(w - 3)$ 27. $(3x + 2)(2x - 3)$

28. $(3x^2y^2 + 1)(2x^2y^2 + 5)$ 29. $(2a - 3)(7a + 5)$

30. $(5p + 3)(3p - 2)$ 31. $(x + y)(x^4 + x^3y + x^2y^2 + xy^3 + y^4)(x - y)$
$(x^4 - x^3y + x^2y^2 - xy^3 + y^4)$

32. $(3 - 2x)(81 + 54x + 36x^2 + 24x^3 + 16x^4)$

II. 1. $-2, 5$ 2. $-\frac{1}{2}, \frac{2}{3}$ 3. $-1, \frac{3}{2}$ 4. $0, \pm 5$

III. 1 $\frac{x + 3}{2}$ 2. $-\frac{7a^2b}{c + b}$ 3. $\frac{a^2 + ab + b^2}{3}$ 4. $x + 4$ 5. $\frac{1}{3}$

6. $(x^2 - xy + y^2)(x - 3y)$