Definitions:

Linear function: a funtion f is a linear function provided there exist real numbers b and m such that for every x in the domain of f,

$$f(x) = mx + b$$

(Remember the old slope-intercept form: (y=mx+b)?)

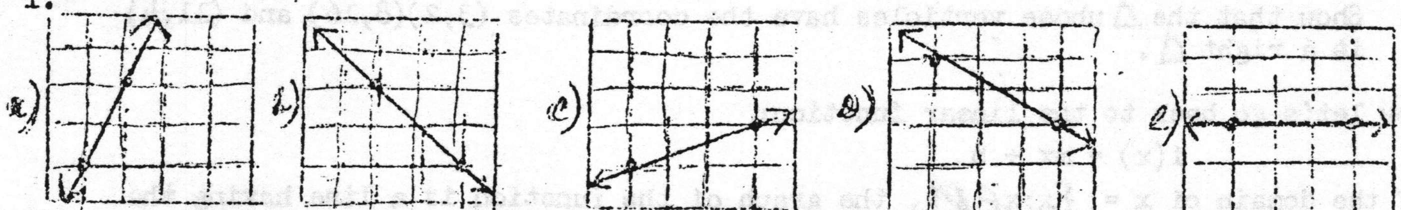
SLOPE of a line: If (x_1, y_1) and x_2, y_2) are coordinates of any two points on a non-vertical line, then slope can be given by:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$\left(\frac{\Delta y}{\Delta x}\right)$ $\left(\frac{rise}{run}\right)$ (L to R)

EXERCISE I

1.



Find the slope of the functions in each of the graphs above.

2. Find the slope of the function having these two points in two ways:
   (1) plot the points, draw a line connecting the two points and determine the slope of the line by inspection.
   (2) solve for m algebraically

a) (2,4);(5,7)
b) (-2,3),(4,5)
c) (2,1),(-3,4)
d) (-3,4),(2,1)
e) (-3,-1),(-1,-3)
f) (-2,5),(0,4)
g) (-2,3),(5,3)
h) (4,5),(-2,-1)

3. Why could we not include a line containing the points (3,-2), and (3,5) in question #2?

4. What do you know about the monotonicity of a linear function having,
   (a) a positive slope?
   (b) a negative slope?

More definitions:

Two non-vertical lines are parallel if they have the same slope.

The converse of this is also true: (If 2 lines have the same slope then the two lines are parallel)

Two non-vertical lines are perpendicular if the product of their slope is -1.

(We call these negative reciprocals)
the converse also true.

EXERCISE II

1. Find the slope of all lines:
   (1) parallel to the line containing these pairs of points, and
   (2) perpendicular to this line

   a) (3,5)(4,7) c) (-2,-1)(-3,-4)

   b) (-2,3)(4,2) d) (3,-2)(5,3)

2. Classify the lines determined by the two pairs of points as parallel, perpendicular, or neither one.

a) (-2,7),(3,6) and (4,2),(9,1) b) (0,0),(-5,3) and (5,2),(0,5)

c) (2,5),(8,7) and (-3,1),(-2,-2) d) (5,3),(-5,-2) and (6,-2),(4,5)

3. Show that the △ whose verticles have the coordinates (3,2)(8,16) and (11,4) is a right △.

Now let's go back to the linear function:

$f(x) = mx + b$

If the domain of x = $\{x : x \in \mathbb{R}\}$, the graph of the function is a line having the slope m and intersecting the y-axis at the point (0,b)

$y = mx + b$, when $m \neq 0$
m = slope
b = y-intercept

EXERCISE III

1. Find the slope of the graph of these functions and
2. Determine the point at which the line would cross the y-axis.

   a) $y = 3x + 5$ c) $2x + 3y = 9$

   b) $y = -x - 4$ d) $3x - 2y = 12$

   e) $7x - 3y - 4 = 0$

3. Graph the functions having the given slopes and y-intercepts. (Use graph paper)

   a) $m = 2$ $b = 1$ d) $m = -\frac{3}{8}$ $b = 5$

   b) $m = \frac{2}{5}$ $b = -3$ e) $m = 1$ $b = -2$

   c) $m = -3$ $b = 2$ f) $m = 0$ $b = 4$

4. Give the equations of each of the functions in #3.
5. Graph these functions by using the slope and y-intercept of the equation. (Do not plot any points other than the y-intercept and the point found from the slope).

   a) $y = 3x + 2$ d) $2x + 3y = 9$

   b) $2y = 7x + 6$ e) $5x - 2y = 8$

   c) $x + y = 7$ f) $3y = 5x$

6. Find the equations of the lines meeting these conditions:
   a) y-intercept = 4 ; parallel to the graph of $2y-4x=7$
   b) y-intercept = -2; perpendicular to the graph of $3y=9x+2$
   c) y-intercept = 7 ; parallel to the graph of $6x+3y=4$
   d) y-intercept = -1; perpendicular to the graph of $x-4y=13$
   e) passing through (3,-1) ; parallel to the graph of $2x+4y=7$
   f) passing through (4,3) ; perpendicular to the graph of $8x+4y=11$

Special Cases of linear functions:

A. Constant function - If $m=0$, the polynomial $y=mx+b$ is equivalent to b. The function f with $f(x)=b$ is the constant function. If its domain is $\mathbb{R}$, the graph of f is a horizontal line through the point (o,b).

B. Direct variation - (also called the multiplying function) - the special case of the linear function $y=mx+b$ when b=o. The function $y=mx$ expresses direct variation, that is y varies directly with x (or y is directly proportional to x) because the ratio $\frac{y}{x}$ is a nonzero constant. This constant, called the constant of variation or the constant of proportionality or a parameter can be represented by m or, as is most frequently done, by k.

Let f be the function whose ordered pairs (x,y) are given in this table. Observe that the quotient, or ratio $\frac{y}{x}$ for every pair of numbers is the same.

x	y	$\frac{y}{x}$
1	$-\frac{1}{4}$	$-\frac{1}{4}$
4	-1	$-\frac{1}{4}$
8	-2	$-\frac{1}{4}$
-12	3	$-\frac{1}{4}$

$\frac{y}{x} = -\frac{1}{4}$ or $y = -\frac{1}{4}x$

the constant of variation is $-\frac{1}{4}$ ($k=-\frac{1}{4}$)

The graph of every direct variation with domain $\mathbb{R}$ is a line that has slope m and passes through the origin.

NOTE: the constant function + the direct variation = the linear function.

$$f(x) = b$$
$$g(x) = mx$$
$$[f+g](x) = mx + b$$

EXERCISE IV

1. Graph these constant functions:
   a) $y=3$ b) $y=-2$
2. Given that p varies directly as q and that the value of p is 5 when the value of q is 10, find:
   a) the constant of variation
   b) the value of p when 28 is the value of q

3. State whether these funtions are linear or not. If thé funtion is leinear, is it a direct variation?

a) $f(x) = 2x + 1$ d) $f(x) = 1/x$

b) $f(x) = 3/4x$ e) $f(x) = 5$

c) $f(x) = 2x^2$ f) $f(x) = -x$

4. Assume that y varies directly as x
a) If the value of y is 120 when the value of x is 40, find the value of y when x has the value of 13.

b) If y is 6 when x is 42, find the value of x when y is 17.

5. Show that the set of ordered pairs

$$\{(4,-2), (5,-2\tfrac{1}{2}), (6,-3), (7,-3\tfrac{1}{2}), (8,-4)\}$$

is an example of direct variation. Find the constant of variation,

Definition: A zero of a funtion is any number in the domain which makes the value of the funtion equal to zero

To find a zero of the function, set $f(x) = 0$ and solve for x.

Example: Given that $f(x) = 3x-12$, find a zero of the funtion.

Solution:

$$f(x) = 3x-12$$

$$3x-12 = 0$$
$$3x = 12$$
$$c = 4$$

Check: $f(4) = 3(4)-12 = 0$ ✓

EXERCISE V

1) Find a zero of these function:

a) $f(x) = 5x+2$ b) $f(x) = -2x + 4$

2) Find the value of the function for each element of the given set to determine which of the elements are zeros of the function.

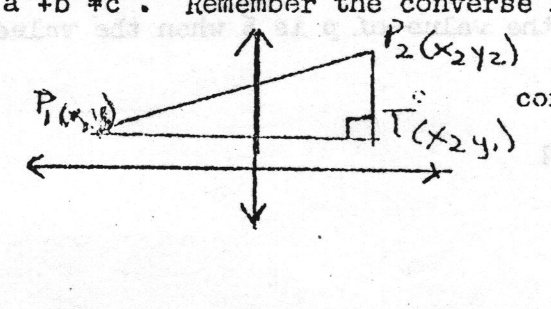
a) $f(x) = x+5$ $\{5,0,-5\}$
b) $f(x) = 3x-7$ $\{7,0,2,7/3\}$
c) $f(x) = 2x-16$ $\{0,2,8\}$

3) How do you determine the x-intercept of a line? What is its relationship to what we're doing here?

-- THE DISTANCE FORMULA --

Let's review the Pythagorean Theorem:

In a right triangle, the square of the length c of the hypotenuse is equal to the sum of the squares of the lengths a and b of the other two sides: $a^2+b^2=c^2$. Remember the converse is also true.



To find the distance between P_1 and P_2 we construct a right triangle with these points: P_1, P_2 and the third vertex of the $\triangle$, T, has the coordinates (x_2,y_1). Since $\overline{P_1T}$ and $\overline{P_2T}$ are parallel to the coordinate axes, their lengths are $|x_2-x_1|$ and $|y_2-y_1|$, respectively. Then, by the Pythagorean Theorem, $d(P_1,P_2)$ (read "the distance from P_1 to P_2") satisfies

$$[d(P_1,P_2)]^2 = |x_2-x_1|^2 + |y_2-y_1|^2$$
$$= (x_2-x_1)^2 + (y_2-y_1)^2$$

Since distance is a nonnegative number.

DISTANCE FORMULA:

$$d(P_1,P_2) = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

THE MIDPOINT FORMULA -- The coordinates of the midpoint $M(x,y)$ of the line segment with endpoints $P_1(x_1,y_1)$ and $P_2(x_2,y_2)$ are

$$x = \frac{x_1+x_2}{2} \text{ and } y = \frac{y_1+y_2}{2}$$

EXERCISE VI

1) In each of these exercises, find
   a) the length of the line segment with endpoints as given and
   b) the coordinates of the midpoint of the segment.
   Express all radicals in simple form.

a) (2,-1) and (5,3)

b) (3,-5) and (4,6)

c) (3,5) and (-1,5)

d) (4,6) and (4,22)

e) (-1/2,6) and (-1/2,-4)

f) (1,2) and (7,10)

g) (3,-1) and (4,-3)

h) $(\sqrt{2},1)$ and $(3\sqrt{2},2)$

INVERSE OF A FUNCTION

A. Every linear function has an inverse except where $m = 0$.

B. A linear function and its inverse both graph symmetric with respect to $y = x$. If you graph a function and its inverse and connect any two corresponding points [(x,y) and (y,x)], the midpoint of that segment will be a point on the graph of $y = x$.

C. If ~~a linear~~ function is ~~monotonic increasing~~, its inverse is also increasing. The same holds ~~true for decreasing~~ functions. (~~These also hold true for all~~ functions.

EXERCISE VII

1. Find the inverses of these functions:

a) $y = 2x+5$

b) $y = 3x+10$

c) $y = -2x+5$

d) $y = 6x-4$

e) $y = 7x$

2. a) Graph the function $y = 2x + 1$
   b) Graph its inverse on the same coordinate plane
   c) Draw 3 line segments, connecting corresponding points.
   d) Draw the $y=x$ axis of symmetry in the same plane. Does it bisect your three line segments from (c)?

3. Check the monotonicity of these functions and their inverses.
   a) $y = 3x + 4$
   b) $y = 4x - 2$
   c) $y = -3x + 1$
   d) $y = -x - 5$

BEHAVIORAL OBJECTIVES

1. Define; linear function
   slope
   zero of a function
2. Given a linear equation:
   a. determine its slope
   b. determine its y-intercept
   c. determine the zero of the function
   d. graph the line
   e. write the equation of a line parallel to the given line
   f. write the equation of a line perpendicular to the given line
3. Describe the relationship between the slopes of:
   a. parallel lines
   b. perpendicular lines
4. Write the equation of a line when given:
   a. two points on the line
   b. y-intercept and the slope
   c. any point and the equation of a line parallel or perpendicular to the line
5. Write the equation for direct variation.
6. Given a direct variation equation, determine
   a. its graph
   b. the constant of proportionality
   c. the value of a coordinate of any point when given the other coordinate and the constant of variation
7. Given two points, find
   a. the distance between the two points
   b. the midpoint of that segment
8. Given a linear function, determine
   a. its range
   b. its domain
   c. its inverse
   d. the graph of the function and its inverse

When this L.A.P. is finished,
1. Check your behavioral objectives - can you do them all?
2. Take the Trial Run
3. Take the test.

LINEAR FUNCTIONS ANSWERS

EXERCISE I

1. a) 2 b) -1 c) 1/3 d) -2/3 e) 0

2. a) 1 b) 1/3 c) -3/5 d) -3/5 e) -1 f) -1/2

g) 0 h) 1 3. Slope is undefined due to division by zero.

4. a) increasing b) decreasing

EXERCISE II

1. a) -2, -1/2 b) -1/6, 6 c) 3, -1/3 d) 5/2, -2/5

2. a) parallel b) parallel c) perpendicular d) neither

3. The side determined by (3,2) and (11,4) has a slope of 1/4. The side determined by (11,4) and (8,16) has a slope of -4. Hence, the lines are perpendicular.

EXERCISE III

1-2. a) 3, 5 b) -1, -4 c) -2/3, 3

d) 3/2, -6 e) 7/3, -4/3

4. a) $y = 2x + 1$ b) $y = 2x/5 - 3$ c) $y = -3x + 2$

d) $y = -3x/5 + 5$ e) $y = x - 2$ f) $y = 4$

6. a) $y = 2x + 4$ b) $y = (-1/3)x - 2$ c) $y = -2x/2 + 7$

d) $y = -4x - 1$ e) $y = -x/2 + \frac{1}{2}$ f) $y = \frac{1}{2}x + 1$

EXERCISE IV

2. a) $\frac{1}{2}$ b) 14

3. a) linear; b) linear, direct variation c) not linear d) not linear e) linear f) linear, direct variation

4. a) 39 b) 119

5. $y = -\frac{1}{2}x$. The constant of variation is $-\frac{1}{2}$.

EXERCISE V

1. a) -2/5 b) 2 c) 2. a) -5 b) 7/3 c) 8

3. Let y = 0 and solve for x. The X-intercept is the zero of the function.

EXERCISE VI.

a) 5, (7/2,1) b) $\sqrt{122}$, (7/2,1/2) c) 4, (1,5) d) 16, (4,14)

e) 10, ($-\frac{1}{2}$,1) (f) 10, (4,6) g) $\sqrt{5}$, (7/2,-2) h) 3, ($2\sqrt{2}$, 3/2)

EXERCISE VII

1. a) $y = (x - 5)/2$ b) $y = (x - 10)/3$ c) $y = (x - 5)/-2$
d) $y = (x + 4)/6$ e) $y = x/7$.

3. a) and b) are increasing.

ALGEBRA 2 — LINEAR FUNCTIONS — TRIAL RUN

I. Define: a) linear function b) slope of a line
c) direct variation d) zero of a function.

II. Give the following information for each of these linear functions:

	$y = 7x - 3$	$2x + 3y = 12$	$3x - 5y = 10$
1. slope of the line			
2. Y-intercept of the line			
3. monotonicity of the line			
4. zero of the function			
5. inverse of the function			
6. monotonicity of f^{-1}			
7. X-intercept of the original line			
8. Slope of the line parallel to the graph of f.			
9. Slope of the line perpendicular to the graph of f.			

III. Give the following information for the functions containing the given points:

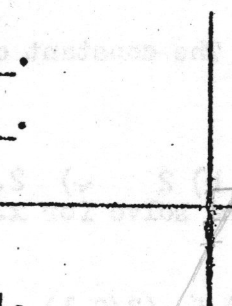
	a) (2,5); (3,8)	b) (-2,4); (-5,-3)
1. the equation of the line:		
2. length of determined segment		
3. midpoint of determined segment		

IV. $f(x) = 3x - 2$. 1. The domain of f is ________.

2. The range of f is ________.

3. $f^{-1}(x) =$ ________

4. Graph f and f^{-1} on the same coordinate system.



5. What symmetry exists between the graphs of f and f^{-1}?

V. 1. Write the equation of the line which has a slope of 5 and Y-intercept 4.

2. Write the equation of the line which has a slope of 3/4 and Y-intercept -2.

VI. Consider the line $\overleftrightarrow{AB}$ with equation $y = 5x - 2$. Write the equation of the line $\overleftrightarrow{CD}$ which contains the point (3,7) and is:

a) parallel to $\overleftrightarrow{AB}$ b) perpendiculat to $\overleftrightarrow{AB}$.

VII. Write the equation for direct variation
a) Where the constant of variation is 17
b) For the linear function having the points (-4,2) and (-2,1).

VIII. Given that p varies directly as q and the value of p is 4 when q is -12,
a) find the constant of variation.
b) find the value of p when the value of q is -14.

ANSWERS:

I. See L.A.P.

II. 1. 7, -2/3, 3/5 2. -3, 4, -2

3. increasing, decreasing, increasing 4. 3/7, 6, 10/3

5. (x + 3)/7, 3x/-2 + 6, (10 + 5x)/3 6. Inc., dec., inc.

7. 3/7, 6, 10/3 8. 7, -2/3, 3/5

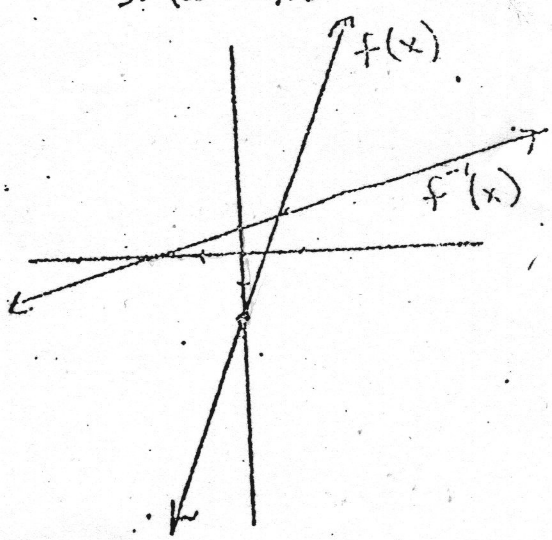
9. -1/7, 3/2, -5/3

III. 1. y = 3x - 1, y = (7x + 26)/3 2. $\sqrt{10}$, $\sqrt{58}$

3. (5/2, 13/2) (-7/2, 1/2)

IV. 1. All real numbers 2. All real numbers 3. (x + 2)/3

3. (x + 2)/3 4. the line x = y



V. a) y = 5x + 4 b) y = 3x/4 - 2

VI. a) y = 5x - 8 b) y = -x/5 + 38/5

VII. a) y = 17x b) y = -x/2

VIII. a) -1/3 b) 14/3