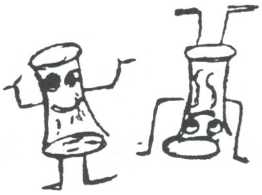
ALGEBRA 2

LEARNING ACTIVITIES PACKAGE

LOGARITHIMS



BEHARIORAL OBJECTIVES

I. Define $\log_b a$ for b a positive number and $b \neq 1$

II. Know and use these important theorems

- A. $\log_b 1 = 0$
- B. $\log_b b = 1$
- C. $\log_b b^x = x$

III. Be able to apply these general theorems for working with logs. Proof is optional.

- A. $\log_b(xy) = \log_b x + \log_b y$
- B. $\log_b(\frac{x}{y}) = \log_b x - \log_b y$
- C. $\log_b x^y = y \log_b x$
- D. $b^{\log_b x} = x$
- E. $\log_b x = \frac{\log x}{\log b}$

IV. Identify

- A. The base of a common logarithm
- B. The characteristic of a common logarithm
- C. The mantissa of a common logarithm

V. Given a number, be able to determine its approximate common logarithm by using a log table.

VI. Given the common log of a number, determine

- A. The characteristic of the log
- B. The mantissa of the log
- C. The antilog

VII. Be able to employ the techniques of interpolation

VIII. Determine roots and powers of given numbers using logs

SECTION I THE DEFINITION OF A LOGARITHM

DEFINITION: $\log_b a = x$ if and only if $b^x = a$, for $b > 0$, and $b \neq 1$.

$\log_b a$ is read: "log to the base b of a equals x."

Note x is the power you must raise b to to get a.

b, the base must be positive. Since b must be positive, it follows that a must be positive too. x can be any real number. If no base is written, it is understood to be ten.

Study these examples of applications of the definition of a logarithm:

1. $\log_2 8 = 3$ (because $2^3 = 8$)
2. $\log_5 25 = 2$ (because $5^2 = 25$)
3. $\log_4 8 = \frac{3}{2}$ (because $4^{3/2} = 8$)
4. $\log 100 = 2$ (because $10^2 = 100$)
5. $\log \frac{1}{10} = -1$ (because $10^{-1} = \frac{1}{10}$)

More Examples:

1. $\log_3 9 = x$	2. $\log_x 8 = 3$	3. $\log_2 x = -4$
$3^x = 9$	$x^3 = 8$	$2^{-4} = x$
$x = 2$	$x = 2$	$\frac{1}{16} = x$

Exercise 1: Solve each of the following for x:

1. $\log \frac{1}{100} = x$
2. $\log_5 625 = x$
3. $\log_{\frac{1}{2}} 4 = x$
4. $\log_4 \frac{1}{2} = x$
5. $\log_9 27 = x$
6. $\log_x 6 = 1$
7. $\log_x 10 = \frac{1}{2}$
8. $\log_x 8 = \frac{3}{2}$
9. $\log_x 25 = -2$
10. $\log_x \frac{1}{8} = -3$
11. $\log_3 x = -2$
12. $\log_4 x = \frac{3}{2}$
13. $\log_{\frac{1}{2}} x = -3$
14. $\log_5 x = 2$
15. $\log_{\sqrt{5}} x = 4$

SECTION II THEOREMS FOR LOGS

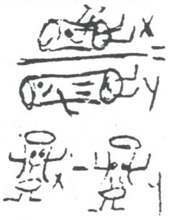
1. **PRODUCT RULE:** The log of a product is the sum of the logs.

$$\boxed{\log_b(xy) = \log_b x + \log_b y}$$

Proof: let $\log_b x = a$ and $\log_b y = c$

Then: $b^a = x$ and $b^c = y$ (Definition of log)

Hence: $\log_b(xy) = \log_b(b^{a+c}) = a + c = \log_b x + \log_b y$



2. **QUOTIENT RULE:** The log of a quotient is the difference of the logs.

$$\boxed{\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y}$$

The proof can be carried out much like the proof for the product rule. Why don't you give it the "old college try?"

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3. **POWER RULE:**

$$\boxed{\log_b x^a = a \log_b x}$$

Proof: Let $\log_b x = c$. Then $b^c = x$

So: $\log_b x^a = \log_b(b^c)^a = \log_b b^{ca} = ca = ac = a \log_b x$

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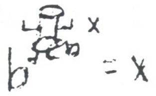
4. **HANDY RULE:**



$$\boxed{b^{\log_b x} = x}$$

Proof: Let $b^{\log_b x} = y$

Take $\log_b$ of both sides: $\log_b b^{\log_b x} = \log_b y$

$\log_b x \log_b b = \log_b y$ (Power R.)

$\log_b x = \log_b y$ ($\log_b b = 1$)

$x = y$

EXAMPLES OF THE USE OF THE THEOREMS:

1. Let $\log x = a$ and $\log y = b$

$\log xy = \log x + \log y$

$= a + b$

2. Let $\log 2 = a$ and $\log 3 = b$

$\log 3/2 = \log 3 - \log 2$

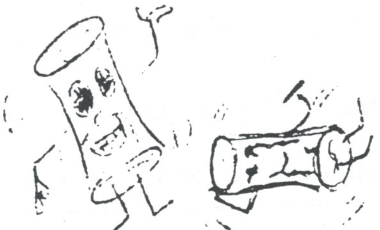
$= b - a$

3. Let log x = a and log y = b

$$\log \frac{x^2}{y} = \log x^2 - \log y$$
$$= 2 \log x - \log y$$
$$= 2a - b$$

4. Let log 2 = a and log 3 = b

$$\log \sqrt{12} = \log 12^{\frac{1}{2}}$$
$$= \tfrac{1}{2} \log 12$$
$$= \tfrac{1}{2} \log(2^2 \cdot 3)$$
$$= \tfrac{1}{2}(\log 2^2 + \log 3)$$
$$= \tfrac{1}{2}(2 \log 2 + \log 3)$$
$$= \tfrac{1}{2}(2a + b)$$
$$= a + \tfrac{1}{2}b$$



"NOW, ROLL YOUR OWN!"

EXERCISE 2

A. Let log x = a and log y = b (Remember, log 10 = 1, log 100 = 2, etc.)

Evaluate:

1. $\log x^4$
2. $\log \frac{x}{y}$
3. $\frac{\log x}{\log y}$
4. $\log \sqrt{xy}$
5. $\log (xy)$
6. $\log (xy^3)$
7. $\log (x^4y^2)$
8. $\log 100x$
9. $\log \frac{100}{y}$
10. $10^{\log x}$

B. Let log 2 = a and log 3 = b (Don't forget log 10 = 1, log 100 = 2, etc.)

1. log 6
2. log 100
3. log 9
4. log 5
5. log 6/9
6. $\frac{\log 6}{\log 9}$
7. log 5,000
8. $\log \sqrt{6}$
9. $10^{\log 2}$
10. log 4800

SECTION III SOLVING EQUATIONS INVOLVING LOGS

This section applies the definition and theorems of logs to solve equations. The theorems are generally worked in <u>reverse</u>; i.e. going from

sum of logs ⟶ log of the product

$$\log x + \log y \longrightarrow \log xy$$
$$\log x - \log y \longrightarrow \log x/y$$
$$a \log x \longrightarrow \log x^a$$

The goal in solving an equation involving logs is to have

1. a single <u>log</u> statement equal a real number

OR. 2. a single log statement equal a single log statement (logs must have same bases.)

<u>Examples</u>: Solve each of the following for x:

1. $\log x + \log 3 = 2$

$\log 3x = 2$ (Product rule)

$3x = 10^2$ (Def. of log)

$\underline{x = \frac{100}{3}}$ (Algebra)

2. $\log_3 x - \log_3 4 = \log_3 8$

$\log_3 x/4 = \log_3 8$ (Quotient rule)

$x/4 = 8$ (Drop logs.)

$\underline{x = 32}$ (Algebra)

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3. $\log_2 \sqrt{\frac{7x - 5}{7}} = 0$

$\sqrt{\frac{4x - 5}{7}} = 2^0 = 1$ (Def. of log)

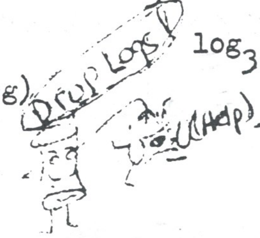
$4x - 5 = 7$ (Algebra)

$4x = 12$

$\underline{x = 3}$

4. $\log_3 x + \log_3(x + 1) = \log_3 2$

$\log_3 x(x + 1) = \log_3 2$



$\rightarrow x^2 + x = 2$

$x^2 + x - 2 = 0$

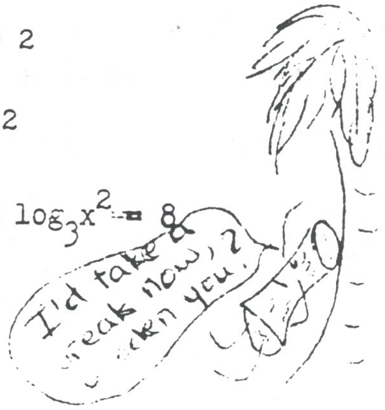
$(x + 2)(x - 1) = 0$

$x = -2$ or $\underline{x = 1}$

<u>REJECT</u>--do not take the log of a negative

<u>EXERCISE 3</u> Solve each of the following for x. Be sure to check all solutions. Do not permit a result which would cause the taking of the log of a negative number.

1. $\log_2 x + \log_2 3 = \log_2 6$
2. $\log_3(x^2 + 3) = 2$
3. $\log_3(x^2 - 27) = 2$
4. $3 \log|x| = 9$
5. $\log_2(x^2 + 3x - 2) = 3$
6. $2 \log x = 10$
7. $\log x = 5 \log 2$
8. $\log\sqrt{x} - \log 3 = \log 2$
9. $\log x + \log 2 = 3$
10. $\log_5 x - \log_5 3 = 2$
11. $\log_5 x - \log_5 3 = \log_5 2$
12. $5 \log x = \log 32$
13. $7^{\log_7 x^2} = 5$
14. $10^{\log(x + 3)} = 10$
15. $2 \log_3 x^2 = 8$



SECTION IV COMMON LOGS

Logarithms base 10 are called common logs.
10 is a convenient base because the number system commonly used is base 10.

Note: $\log_{10} 1 = 0$; $\log_{10} 10 = 1$; $\log_{10} 100 = 2$; etc.

The log tables were constructed with great difficulty by a French mathematician named Napier. See the table in this L.A.P.

When no base is written for a given log, the base is understood to be 10. Since this section concerns itself with base 10 logs, we shall no longer be indicating the value of the base.

LET'S READ THE LOG TABLE!

N	0	1	2	3	4	5	6	7	8	9
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055
.	.	.	.	.	.	.	.	.	.	.

The table is built for numbers written in scientific notation.

Example: $\log 6.23 = .7945$; This means that $10^{.7945} = 6.23$

N, see the table, is some number between 1.00 and 9.99. <u>You need to imagine a decimal point between the two digits!</u> The four digit number in the table needs to be thought of as a number between 0 and 1. <u>You need to imagine a decimal point before each four digit number.</u>

<u>More examples</u>: $\log 62.3 = \log(6.23 \times 10^{1}) = \log 6.23 + \log 10 = .7945 + 1$
$= 1.7945$

$\log .00636 = \log(6.36 \times 10^{-3}) = \log 6.36 + \log 10^{-3} = .8035 - 3$

$\log 62800 = \log(6.28 \times 10^{4}) = \log 6.28 + \log 10^{4} = .7980 + 4 =$
4.7980

The four digit decimal in the table is called the <u>mantissa</u> of the log. The power of ten needed to place the number to be "logged" is called the <u>characteristic</u> of the log.

$$\log 625 = \log(6.25 \times 10^2) = \log 6.25 + \log 10^2 = .7959 + 2 = \underset{\text{characteristic}}{2}.\underbrace{7959}_{\text{mantissa}}$$

EXERCISE 4 For each of the following N, determine log N. Use the log table found in this L.A.P.

1. N = 683	2. N = .00325	3. N = 705,000	4. N = 53.4
5. N = 310,000	6. N = .195	7. N = 8.05	8. N = .0206
9. N = 1,000	10. N = 64.8	11. N = 100	12. N = 4.5×10^{11}

SECTION V ANTILOGARITHMS

When given the log of a number, one can use the table, the knowledge of the roles of the characteristic and the mantissa, and work backward to determine the given number.

Examples: 1. $\log N = \underset{\text{Characteristic power of ten}}{7}.\underbrace{9015}_{\text{Mantissa-- found in the log table}}$

The table shows .9015 as the log of 7.97.

The characteristic 7 tells us to multiply 7.97 by 10^7

Hence, if log N = 7.9015, then N = 7.97×10^7

N = 79,700,000

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2. log N = 2.8921

Use the table to verify:

N = 7.8×10^2

N = 780

3. log N = .5403 - 3

Use the table to verify:

N = 3.47×10^{-3}

N = .00347

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4. log N = -1.5129 (OH! OH! The mantissa is negative!)

= 2 - 1.5129 ÷ 2 (The "handy man's" way to repair a negative mantissa.)

-= .4871 - 2

SO: N = 3.07×10^{-2} = .0307

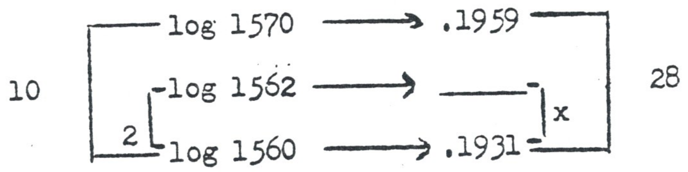
EXERCISE 5 Determine the value of N for each of the following:

1. log N = 3.9643
2. log N = .7033 - 2
3. log N = .6665 - 4
4. log N = 1.8949
5. log N = .2945
6. log N = 2.7459
7. log N = 5.5551
8. log N = .5775 - 5
9. log N = 2.7042
10. log N = 4
11. log N = -2
12. log N = 100
13. log N = -1.6946
14. log N = -2.4134
15. log N = -.2652

SECTION VI INTERPOLATION

Logs are quickly becoming obsolete as a calculation tool. The pocket calculator is far faster and more accurate. Hence, we shall not burden the student of the '70's with long tedious calculations. However, we wish to use logs to introduce a valuable process. This process is called <u>interpolation</u>. Interpolation is used frequently in various areas of mathematics and science.

For example, we wish to find log 1562. Recall, we need to express the number in scientific notation in order to use the table. So: 1562 = 1.562 X 10^3.
The table supplies values for log 1.560 and log 1.570. Log 1.562 has a value between the two values supplied by the table. To approximate the value of log 1.562, direct proportion can be used.



Ignore the characteristic.
Compute the differences

Set up a proportion and solve for the 'difference' x.

$$\frac{2}{10} = \frac{x}{28}$$

$$\frac{56}{10} = x$$

$$6 \doteq x$$ (round off to the nearest whole number.)

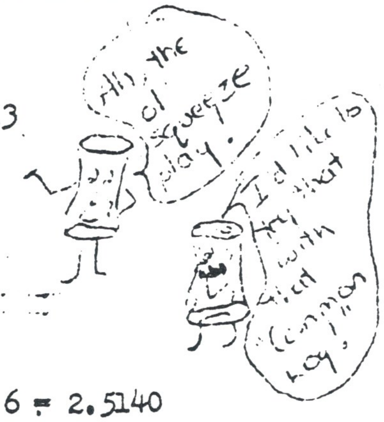
Hence: log 1.562 = .1937 (Add 6 to the lower number.)

log 1562 = 3.1937

Example 2: log N = 2.5140. Determine the value of N.

For now, concentrate only on the mantissa. Find values just larger and just smaller from the table. **SQUEEZE BETWEEN**

$$10\left[\begin{array}{lcl} \text{Log } 3270 & \longrightarrow & 5145 \\ x\left[\begin{array}{l}\log \underline{\qquad} \\ \log 3260\end{array}\right. & \begin{array}{c}\longrightarrow\\ \longrightarrow\end{array} & \left.\begin{array}{r}5140\\ .5132\end{array}\right]8 \end{array}\right]13$$



$$\frac{x}{10} = \frac{8}{13}$$

$$x = \frac{80}{13} \doteq 6$$

Hence, log 3.266 = .5140 So, log 326.6 = 2.5140

Example 3: Interpolation can also be used to approximate the square root of a number. The process is far from accurate but possibly better than guessing. Study the following for $\sqrt{20}$.

$$9\left[\begin{array}{lcl} \sqrt{25} & \text{———} & 5.000 \\ 4\left[\begin{array}{l}\sqrt{20} \\ \sqrt{16}\end{array}\right. & \begin{array}{c}\text{———}\\ \text{———}\end{array} & \left.\begin{array}{r}\underline{\qquad}\\ 4.000\end{array}\right]x \end{array}\right]1$$

$$\frac{4}{9} = \frac{x}{1}$$

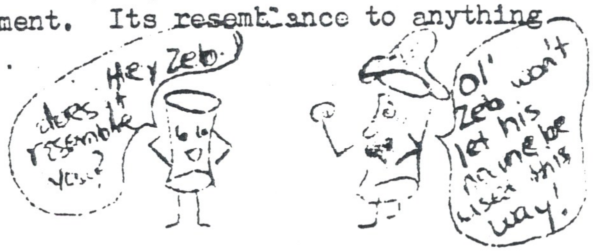
$$x \doteq .444$$

Hence, $\sqrt{20} \doteq 4.444$ (Note: $4.444^2 = 19.749$. Not bad!)

EXERCISE 6 Use interpolation to evaluate each of the following for N:

1. log N = 2.9209
2. log N = .6584
3. log N = 1.2928
4. log N = .3191 - 2
5. log 2.754 = N
6. log 1.036 = N
7. $\sqrt{8}$ = N
8. $\sqrt{30}$ = N
9. $\sqrt[3]{20}$ = N
10. If zeb 8 = .0023 and zeb 9 = .0059, then zeb 8.6 = N
11. If zeb 11 = .0088 and zeb 12 = .0096; then zeb 11.6 = N

(The word 'zeb' is a donation from the management. Its resemblance to anything mathematical is purely coincidental.)



SECTION VII COMPUTATION

Since most products and quotients can be easily computed on the simplest pocket calculator, we will only concern ourselves with "strange" roots and powers of numbers. Many calculators do no more than take the square root of a number. The higher roots are not available.

Example 1: $\sqrt[3]{42} = N$ Solve for N

1. Take the log of both sides: $\log \sqrt[3]{42} = \log N$
2. Use rational exponents: $\log 42^{1/3} = \log N$
3. Power rule for logs: $\frac{1}{3} \log 42 = \log N$
4. Evaluate log 42; $\frac{1}{3}(1.6232) = \log N$
5. Multiply: $.5411 = \log N$
6. Take the antilog(approximate) $3.48 \doteq N$

Check: $(3.48)^3 \doteq 42.14$ (Close enough!)

Example 2: $\sqrt[5]{\frac{2.84}{1.08}} = N$

1. Take the log of both sides: $\log \sqrt[5]{\frac{2.84}{1.08}} = \log N$
2. Power rule for logs: $\frac{1}{5} \log \frac{2.84}{1.08} = \log N$
3. Quotient rule: $\frac{1}{5}(\log 2.84 - \log 1.08) = \log N$
4. Evaluate logs: $\frac{1}{5}(.4533 - .0334) = \log N$
5. Subtract: $\frac{1}{5}(.4199) = \log N$
6. Multiply: $.0840 = \log N$
7. Antilog $1.215 = N$

EXERCISE 7 Evaluate each of the following. You do not need to interpolate. Make healthy guesses when necessary. When checking your answers just make sure that you are in the "ball park." Your answers do not have to agree perfectly.

1. $\sqrt{8}$
2. $\sqrt{20}$
3. $\sqrt[3]{20}$
4. $\sqrt[3]{109}$
5. $(\sqrt{18.7})^3$
6. $(20.8)^{3.2}$
7. $(\sqrt[10]{10})^3$
8. $(\sqrt[7]{218})^3$

ALGEBRA 2 — LOGS TRIAL RUN

I. Solve each of the following for x:

1. $\log_2 x = 5$
2. $\log 0.1 = x$
3. $\log_3 (\frac{1}{27}) = x$
4. $\log_{\sqrt{2}} 4 = x$
5. $\log_2 x = -3$
6. $\log_{2/3} (3/2) = x$
7. $\log_2 x^2 = 6$
8. $\log_2 \sqrt{x} = 2$
9. $\log_2 |x| = -1$

II. Given: $\log 2 = a$; $\log 3 = b$; $\log 7 = c$. Evaluate, in terms of a, b, and c, each of the following:

1. $\log 90$
2. $\log \frac{9}{4}$
3. $\frac{\log 21}{\log 2}$
4. $\log 6^2$
5. $\log 1$
6. $(\log 2)(\log 14)$
7. $\log 42$

III. Solve each of the following for x:

1. $\log 3 + \log x = \log 18$
2. $\log x + \log (x + 15) = 2$
3. $8^{\log_8 x^3} = 27$
4. $\log x^2 - \log 4 = 2$
5. $\log_3(x + 1) + \log_3(x + 3) = 1$
6. $\log_5 \sqrt{\frac{3x + 4}{4}} = 0$
7. $\log \frac{x + 3}{x - 1} = \log 3$
8. $\log_2 |x| = 3$

IV. Use the log table to determine N for each of the following:

1. $\log 18.2 = N$
2. $\log N = 3.4843$
3. $\log .00252 = N$
4. $\log N = .7356 - 2$
5. $\log N = -31238$

V. For each of the following <u>interpolate</u> to find N:

1. $\log 834.3 = N$
2. $\log N = .7216 - 2$
3. $\sqrt{7}$

VI. Do the indicated computations using logs. You need not interpolate.

1. $(26.5)^{.8}$
2. $\sqrt[5]{325}$
3. $(\sqrt{\frac{28}{3}})^5$
4. $(16.2)^{-2}$

ALGEBRA 2 **LOGS TRIAL RUN ANSWERS**

I. 1. 32 2. -1 3. -3 4. 4 5. $\frac{1}{8}$

6. -1 7. ± 8 8. 16 9. $\pm \frac{1}{2}$

II. 1. $2b + 1$ 2. $2b - 2a$ 3. $\frac{b + c}{a}$ 5. 0

6. $a^2 + ac$ 7. $a + b \pm c$ 4. $2a + 2b$

III. 1. 6 2. 5 3. 3 4. ± 20 5. 0

6. 0 7. 3 8. ± 8

IV. 1. 1.2601 2. 3050 3. $.4014 - 3$ 4. .0544 5. .000752

V. 1. 2.9214 2. .05267 3. 2.6

VI. 1. 13.75 2. 3.18 3. 266 4. .00381