PROBABILITY

OBJECTIVES:

I. Review Permutations and Combinations

A. Factorials
B. Permutation Formula $_nP_r$
C. Combination Formula $_nC_r$
D. Determining which to use (order vs. selection)

II. Probability

A. Definition
B. Equally Likely Outcomes
C. Single Stage Experiments
D. Many Stage Experiments
E. Dependent and Independent Events
F. Experiments with or without replacement
G. Odds

SECTION I. PERMUTATIONS AND COMBINATIONS

A **PERMUTATION** of some elements is an ordered arrangement of the elements. Find the permutations:

Think of n=total elements r=# taken at a time

[1] 5 books arranged on a shelf = (5)(4)(3)(2)(1) = 120 (n=5, r=5)
[2] 5 books arranged 3 at a time = (5)(4)(3) = 60 (n=5, r=3)
[3] 5 books arranged 1 at a time = 5 (n=5, r=1)

The general formula for a permutation of n elements taken r at a time is

$$_nP_r = \frac{n!}{(n-r)!}$$

n! is read " n factorial" which is defined as n(n-1)(n-2)(n-3)...(3)(2)(1)
Don't forget that n!= n(n-1)! or n(n-1)(n-2)! etc., 1!=1 and 0! = 1

Ex. 1. for [2] above <5 books arranged 3 at a time> $_5P_3 = \frac{5!}{(5-3)!} = \frac{5!}{2!} = \frac{(5)(4)(3)(2!)}{2!} = (5)(4)(3) = 60$

Ex. 2. How many ways can the letters of the word RUMBLE be arranged ?
There are 6 different letters, we are arranging them 6 at a time
So $_6P_6 = \frac{6!}{(6-6)!} = \frac{6!}{0!} = 6! = (6)(5)(4)(3)(2)(1) = 720$
[In general, $_nP_n = n!$]

Ex. 3. Ten runners enter a race. How many possible ways could they finish if prizes were awarded to 1st, 2nd and 3rd place ?
$_{10}P_3 = \frac{10!}{(10-3)!} = \frac{10!}{7!} = \frac{(10)(9)(8)(7!)}{7!} = (10)(9)(8) = 720$

Ex. 4. ARRANGEMENT WITH REPETITION: How many ways could the letters of the word ARRANGE be arranged ? If all 7 letters were different, it would be 7!, but 2 of the letters appear twice, so divide the result by the duplicates (# of repeats)! = $\frac{7!}{}$ $= \frac{(7)(6)(5)(4)(3)(2!)}{2!\ 2!}$ = 7 6 5 2 3 = 1260
[2 letters repeat twice each--->]

A **COMBINATION** of some elements is any selection of the elements in which order is not important. [Think of members of a TEAM (without ranking or position) such as study team, or selecting 3 books from 5 to be read]

The general formula for a combination of n elements taken r at a time is:

$$_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

[2 notations: $_nC_r$ and $\binom{n}{r}$ mean the same thing]

Ex. 5. How many different 3-person sensitivity groups can be selected from 10 caring individuals?

$$_{10}C_3 = \frac{10!}{3!(10-3)!} = \frac{(10)(9)(8)(7!)}{3!\ 7!} = \frac{(10)(9)(8)}{(3)(2)(1)} = (10)(3)(4) = 120$$

Ex. 6. A final group of 3 needs to be selected from 9 "Jeopardy!" contestants . How many possible combinations ?

$$_9C_3 = \frac{9!}{3!(9-3)!} = \frac{9!}{(3!)(6!)} = \frac{(9)(8)(7)(6!)}{(3!)(6!)} = \frac{(9)(8)(7)}{(3)(2)(1)} = (3)(4)(7) = 84$$

CHOOSING THE USE OF PERMUTATIONS VS. COMBINATIONS :

Use PERMUTATIONS when ORDER IS IMPORTANT.
Use COMBINATIONS when ORDER IS NOT IMPORTANT.

EXERCISE 1

EVALUATE:

1. $_5P_3$ = ________ 2. $_6P_5$ = ________ 3. $_{10}P_4$ = ________

4. $_7C_3$ = ________ 5. $_7C_4$ = ________ 6. $_7C_7$ = ________

CHOOSE P (for permutation) or C (for combination) then solve

7. number of ways 10 semi-finalists can finish in Miss America (5 places: 1st, 2nd, 3rd, 4th, 5th)

8. the number of 2-member committees that can be formed from a group of 12 people

9. the number of 5-letter "words" that can be formed from 9 different letters [we consider a "word" any grouping of letters, not necessarily a word that is meaningful in some language]

10. the number of possible 9-man lineups for the Marlins that can be formed from 20 players (positions: pitcher, catcher, 1B, 2B, 3B, SS, RF, CF, LF)

11. the number of "words" that can be formed from ALABAMA (using all letters)

12. Describe the relationship between $_nC_r$ and $_nC_{n-r}$.

13. NMB's Mu Alpha Theta competition squad needs a 5-member team. How many possible teams can be formed from a total membership of 20 people?

14. How many ways can a HR representative and an alternate be elected from a homeroom of 25 people?

SEE YOUR TEACHER FOR ADDITIONAL ...

SECTION II. **PROBABILITY**

Definition: If an experiment can result in any one of n different, equally likely outcomes , and if exactly m of these outcomes correspond to event A then the probability of A is $P(A) = \frac{m}{n}$

Probability is a RATIO such that $0 \leq P \leq 1$ [with 0 indicating an impossible event and 1 indicating a certain event]

Think of tossing a coin : total possible outcomes = 2 (Heads or Tails), equally likely . The probability of Heads = 1/2 , the probability of tails is also 1/2 .

Rolling one die (singular of "dice") has 6 equally likely outcomes (1,2,3,4,5,6) . P(5) would be 1/6 .

P(#>2): the event is greater than 2, # outcomes that corresponds to >2 would be 4 (a 3,4,5, or 6) so P(#>2) = 4/6 or 2/3

P(9) = 0 P(#<7) = 1

Drawing 1 card from a deck: (a deck has 52 cards: 13 cards per suit (diamonds, hearts, clubs, spades); cards are 2 through 10, Jack, Queen, King, and Ace (in each suit); diamonds & hearts are red, spades and clubs are black)

P(Queen of Hearts) = 1/52 P(a Queen) = 4/52 = 1/13

P(any Diamond) = 13/52 P(a red card) = 26/52 = 1/2

The above are all **single stage experiments** .

For **MULTI-STAGE EXPERIMENTS** (flipping 2 or more coins, tossing 2 or more dice, drawing more than 1 card from a deck {with or without REPLACEMENT}, we need to consider whether the events are DEPENDENT or INDEPENDENT.

INDEPENDENT EVENTS: events are independent if the occurrence of one event DOES NOT affect the other... (such as flipping 2 or more coins, tossing several dice)

Probability that 2 or more independent events will happen is the PRODUCT of their separate probabilities.

Ex1. tossing 2 coins and having both turn-up heads = (1/2)(1/2) = 1/4

Ex2. tossing 2 dice and getting a sum of 6:
here we must find all possible outcomes that satisfy each event(sum)

	2nd-die	1	2	3	4	5	6	
1st die	1	2	3	4	5	6	7	
	2	3	4	5	6	7	8	
	3	4	5	6	7	8	9	[SUMS]
	4	5	6	7	8	9	10	
	5	6	7	8	9	10	11	
	6	7	8	9	10	11	12	

P(sum of 6) = $\frac{\text{\# ways to roll a 6}}{36} = \frac{5}{6}$

[Think of the sum of SIX as the EVENT, which has 5 outcomes (sums of 6) that are FAVORABLE] There are 36 TOTAL POSSIBLE OUTCOMES for 2 dice rolled (see above chart representing 1st and 2nd die and their corresponding SUM). The roll of the 1st die does NOT affect the roll of the second die (each die still has 6 equally likely outcomes).

Ex3. drawing more than 1 card from a 52-card deck WITH replacement. If after each draw, the card is replaced and the deck is shuffled, the successive draws are INDEPENDENT of that 1st draw.

The probability of drawing 2 Aces (WITH replacement) would be [the product of P(Ace) and P(Ace) = (1/13)(1/13) = 1/169

DEPENDENT EVENTS: Two or more events are DEPENDENT if the occurrence (or non-occurrence) of one event DOES affect the probabilities of the others.

If P_1 is the probability of the 1st event, P_2 the probability that after the 1st event has happened, the second will occur, P_3 the probability that after the 1st AND second have happened, the 3rd will occur,etc. then the probability that all events will happen in a given order is $(P_1)(P_2)(P_3)\ldots$

Ex.4 The probability of drawing 2 Aces WITHOUT REPLACEMENT :

$P_1 = 1/13$ $\qquad P_2 = 3/51 = 1/17$

[after 1 Ace has been drawn, there remain only 3 Aces and 51 total cards]

P(2 Aces without replacement) = (1/13)(1/17) = 1/221

MUTUALLY EXCLUSIVE EVENTS: The occurrence of any one event EXCLUDES the occurrence of the others.

The PROBABILITY OF MUTUALLY EXCLUSIVE EVENTS is the SUM of the probabilities of the individual events.

Ex.5 One bag contains 4 white balls and 2 black balls. Another bag contains 3 white balls and 5 black balls. If one ball is drawn from each bag, what is the probability of drawing 1 white ball and 1 black ball.

P(1st ball:white 2nd ball:black) = (4/6)(5/8) = 5/12

P(1st ball:black 2nd ball:white) = (2/6)(3/8) = 1/8

Since these events are MUTUALLY EXCLUSIVE the P = 5/12 + 1/8 = 13/24

ODDS: The ODDS in favor of an event happening is $\dfrac{\#\text{ favorable outcomes}}{\#\text{ unfavorable outcomes}}$

The ODDS AGAINST an event happening is $\dfrac{\#\text{ unfavorable outcomes}}{\#\text{ favorable outcomes}}$

The odds of rolling a 6 with two dice = 5/31 (note: before reducing the fraction the # favorable + # unfavorable = total possible outcomes)

The odds against rolling a 6 would be 31:5 or 31/5

EXERCISE 2

Consider a standard 52-card deck. Find the probabilities for:

1. Drawing one RED card ______ 2. Drawing a FACE CARD (A,K,Q,J)________

3. Drawing 5 or a 6 ______ 4. Drawing a red Jack ______

Consider drawing MORE THAN 1 card WITH REPLACEMENT :

5. P(2 Aces) = __________ 6. P(2 Red cards) = __________

7. P(3 Face Cards) = ________ 8. P(3 Kings) = __________

Consider drawing MORE THAN 1 card WITHOUT REPLACEMENT :

9. P(2 Aces) = __________ 10. P(2 Red cards) = __________

11. P(3 Face Cards)=__________ 12. P(3 Kings) = __________

13. If 3 (six-sided standard) dice are tossed, what is the total number

14. Nine tickets, numbered from 1 to 9, are in a box. If 2 tickets are drawn at random, determine the probability P that:
(a) both are odd ________ (b) both are even ________

(c) one is odd and one is even ________

15. Two dice are rolled :
(a) What are the ODDS in favor of rolling a 7 ________

(b) What are the ODDS against rolling a 7 ________

(c) What is the probability P of rolling a 7 ________

(d) What is the probability of NOT rolling a 7 ________

(e) How does the answer for (c) compare to the answer for (d)?

16. Florida's CASH-3 daily drawing consists of drawing 3 balls, numbered 0 to 9 from 3 (INDEPENDENT) tubes.
(a) What is the probability of drawing 2-3-4 ________

(b) What is the probability of drawing 3-3-3 ________

(c) What is the probability of drawing 2-3-4 in ANY ORDER (2-3-4, or 2-4-3 , or 3-2-4, or 3-4-2, or 4-2-3, or 4-3-2)

____

SEE YOUR TEACHER FOR ADDITIONAL WORKSHEETS

There is NO TRIAL RUN for this L.A.P., review problems in the L.A.P. and the worksheets.

ANSWERS TO EXERCISES :

EXERCISE 1: 1. 60 2. 720 3. 5,040 4. 35 5. 35 6. 1 7. 30,240

8. 66 9. 15,120 10. 60,949,324,800 11. 210 12. same result

13. 15,504 14. 600

EXERCISE 2: 1. 1/2 2. 4/13 3. 2/13 4. 1/26 5. 1/169 6. 1/4

7. 64/2197 8. 1/2197 9. 1/221 10. 25/102 11. 28/1105

12. 1/5525 13. 216 14. (a) 5/18 (b) 1/6 (c) 5/9

15. (a) 1:5 (b) 5:1 (c) 1/6 (d) 5/6 (e) P(A) = 1 - P(not A) - probability of event A happening is 1 - probability of A not happening

16. (a) 1/1000 (b) 1/1000 (c) 3/500