

RELATIONS

Behavioral Objectives

- I. Define
 - A. Relation
 - B. Domain of a relation
 - C. Range of a relation

- II. Given a relation
 - A. Graph the relation
 - B. Determine its domain
 - C. Determine its range

- III. Given two sets of numbers A and B
 - A. Determine
 1. $A \times B$
 2. $B \times A$
 - B. Graph $A \times B$
 - C. Graph $B \times A$
 - D. Determine the domain and range of $A \times B$
 - E. Determine the domain and range of $B \times A$

Def: A relation is a set of ordered pairs.

ex: (1) $\{(2,8), (4,7), (3,9)\}$
 or (2) $\{(x,y): y = 3x+1\}$
 (3) $\{(x,y): y = x^2\}$

Def: The domain of a relation is the set of first components of the ordered pairs of the relation.

ex: (1) $D_{(1)} = \{2,4,3\}$
 (2) $D_{(2)} = \text{all reals}$ or $\{x:x=x\}$ or $(-\infty, \infty)$
 (3) $D_{(3)} = \text{all reals}$ or $\{x:x=x\}$ or $(-\infty, \infty)$

[For (1) and (2) the domain was listed in three acceptable ways. You may choose the method you understand most clearly.]

Def: The range of a relation is the set of second components of the ordered pairs of the relation.

ex: (1) $R_{(1)} = \{8,7,9\}$
 (2) $R_{(2)} = (-\infty, \infty)$
 (3) $R_{(3)} = [0, \infty)$ or $\{y:y \geq 0\}$

Relations

(2)

Notation: for domains and ranges

$$[2, 4] = \{x: 2 \leq x \leq 4\}$$

$$(2, 4] = \{x: 2 < x \leq 4\}$$

$$(2, 4) = \{x: 2 < x < 4\}$$

$$(-\infty, 7] = \{x: x \leq 7\}$$

$$(-\infty, \infty) = \{x: x = x\}$$

$$[7, \infty) = \{x: x \geq 7\}$$

$$(7, \infty) = \{x: x > 7\}$$

Cross Product: If $A = \{2, 3\}$ and $B = \{3, 4, 5\}$ then

$$A \times B = \{(2, 3), (2, 4), (2, 5), (3, 3), (3, 4), (3, 5)\}$$

$$B \times A = \{(3, 2), (3, 3), (4, 2), (4, 3), (5, 2), (5, 3)\}$$

Note: $A \times B$ is a relation.

The domain of $A \times B$ is A

The range of $A \times B$ is B .

For this L.A.P. you will be expected to become an expert at graphing, locating domains and locating ranges.

Relations - Exercises 1

1. Which of the following sets are relations

a) $\{(1, 2), (2, 3), (3, 4)\}$

c) $\{1, 2, 3, 4\}$

b) $\{(1, 2), (2, 2)\}$

d) $\{(1, 2, 3), (2, 3, 4), (3, 4, 5)\}$

2. Let $U = \{0, 1, 2, 3\}$ $U = \{0, 1, 2, 3\}$

a) $U \times U$

b) List the ordered pairs in each of the following relations from $U \times U$ and graph the relation

$$R_1 = \{(x, y) \mid y = x\}$$

$$R_4 = \{(x, y) \mid x + y = 1\}$$

$$R_2 = \{(x, y) \mid y > x\}$$

$$R_5 = \{(x, y) \mid x^2 + y^2 = 1\}$$

$$R_3 = \{(x, y) \mid y < x\}$$

$$R_6 = \{(x, y) \mid 2x = y\}$$

3) Shade in the region of the plane which contains the graph of a relation with domain D and range R .

a) $D = [-1, 1], R = [3, 8]$

b) $D = [0, \infty), R = \{5\}$

c) $D = (-\infty, 0], R = [0, \infty)$

d) $D = \{2, 3\}, R = [-2, 3]$

e) $D = [-1, 0], R = (1, 4)$

f) $D = (-\infty, \infty), R = [1, 1)$ (1, 1)

4. Each of the following represents a relation. Graph the relation and state the domain and the range.

a) $y = 2x + 1$

line

f) $y = |x + 1|$

V shape

b) $y^2 = x$

parabola

g) $x^2 < y^2$

Intersecting lines--
shading

c) $y < 2x + 1$

line-shading

h) $x^2 + y^2 = 1$

circle

d) $2x + 2y = 3$

line

i) $|x| + |y| \leq 1$

line--shading

e) $x = 7$

line

j) $y = \sqrt{x^2 - 9}$

part of a parabola

l) $|x| + |y| = 4$

k) $xy = 1$

hyperbola

5. Graph each of the following relations. State the domain and range for each.

1. $y = x$

2. $y > x$

3. $y \geq x$

4. $y < x$

5. $y = x^2$

6. $y = x^3$

7. $y = 4x$

8. $y = 2x + 1$

9. $y^2 = x$

10. $y < 2x + 2$

11. $y = 7$

12. $x = 7$

13. $y = |x - 1|$

14. $y = -2$

15. $2x - 2y = 3$

16. $x^2 = y^2$

17. $x^2 + y^2 = 25$

18. $4x^2 + 9y^2 = 36$

19. $|x| + |y| = 4$

20. $|x| = |y|$

21. $y = \sqrt{x}$

22. $y = x^4$

23. $y = \sqrt{x^2 - 4}$

24. $y = \frac{1}{x}$

25. $-3x^2 = y$

26. $y = \frac{1}{\sqrt{x^2 - 4}}$

27. $y = (x + 3)^2 - 2$

28. $y \leq (x + 3)^2 + 2$

29. $y \geq (x - 3)^2 + 1$

30. $y = -(x - 3)^2 + 3$

RELATIONS L.A.P. ANSWERS

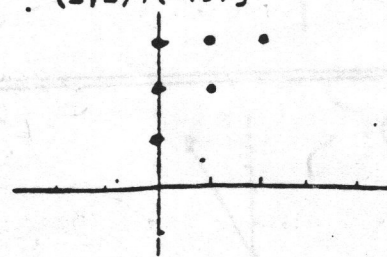
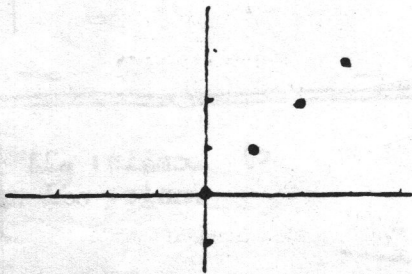
Exercises

1. Only a) and b) are relations

2. a) $\{(0,0), (0,1), (0,2), (0,3), (1,0), (1,1), (1,2), (1,3), (2,0), (2,1), (2,2), (2,3), (3,3), (3,1), (3,2), (3,3)\}$

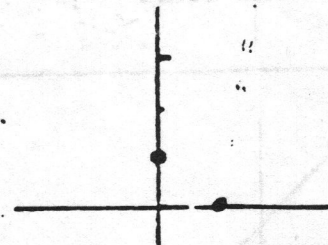
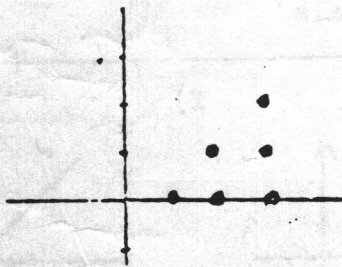
b) $R_1 = \{(0,0), (1,1), (2,2), (3,3)\}$

$R_2 = \{(0,1), (0,2), (0,3), (2,3), (1,2), (1,3)\}$



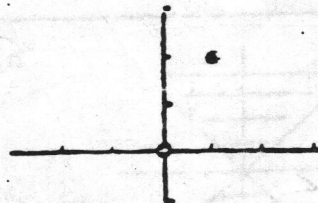
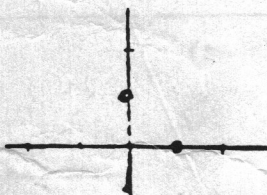
$R_3 = \{(1,0), (2,0), (3,0), (3,2), (3,1), (2,1)\}$

$R_4 = \{(0,1), (1,0)\}$



$R_5 = \{(0,1), (1,0)\}$

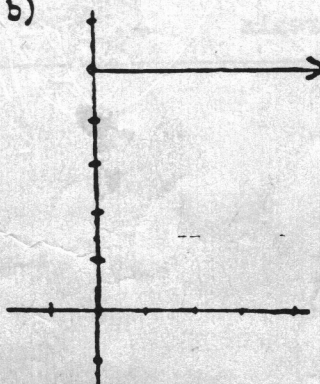
$R_6 = \{(0,0), (1,2)\}$



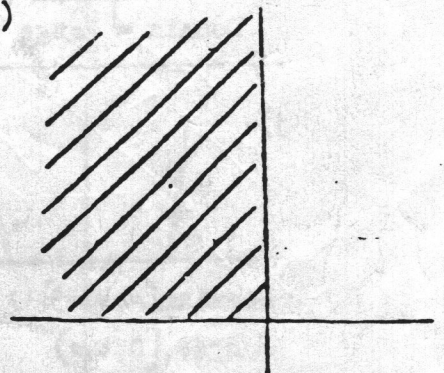
3. a)



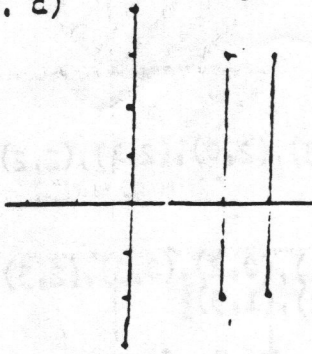
b)



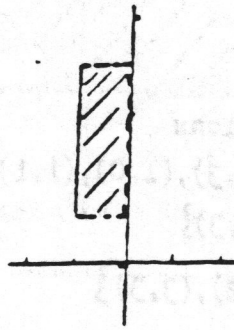
c)



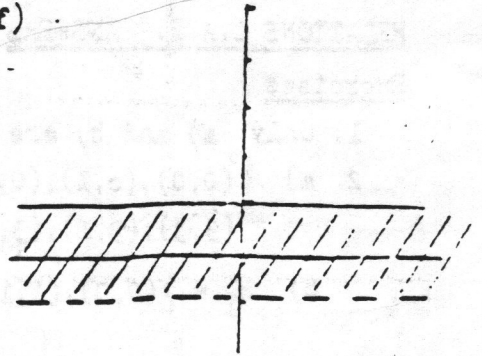
3. d)



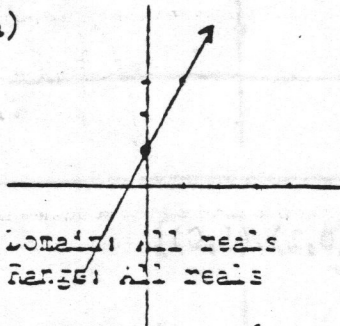
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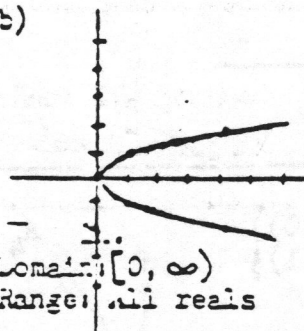
f)



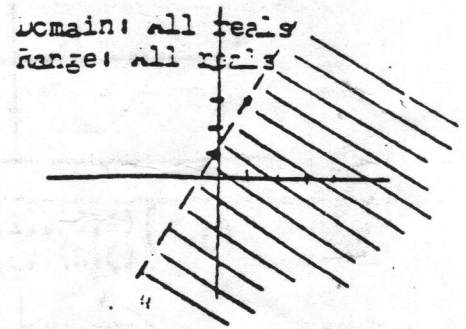
4. a)



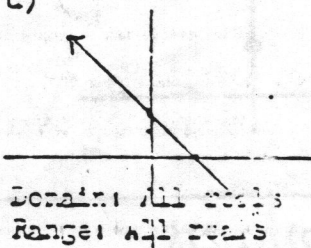
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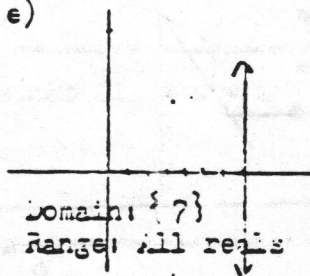
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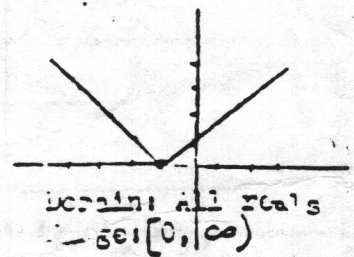
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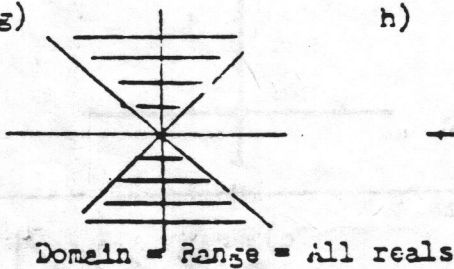
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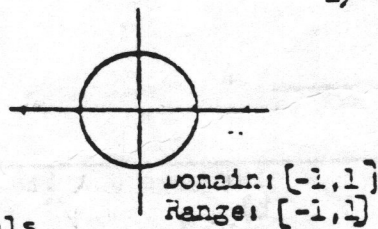
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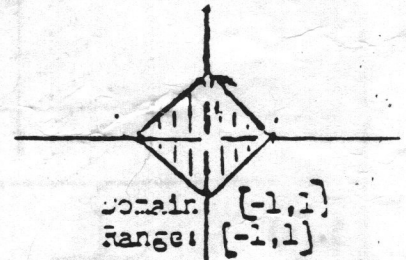
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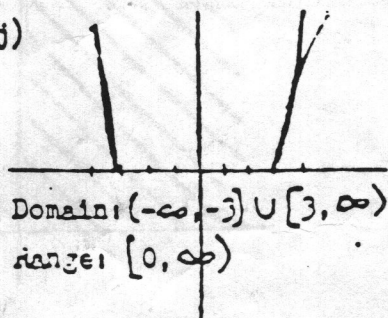
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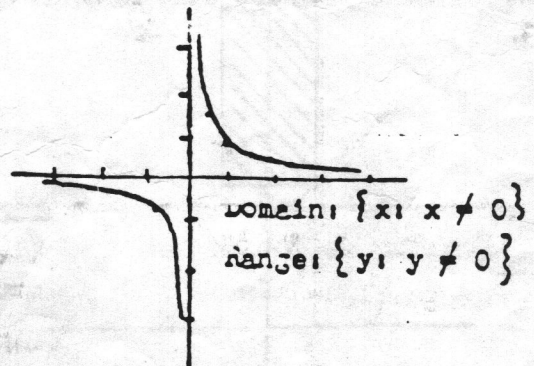
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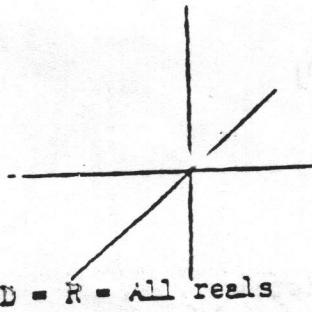
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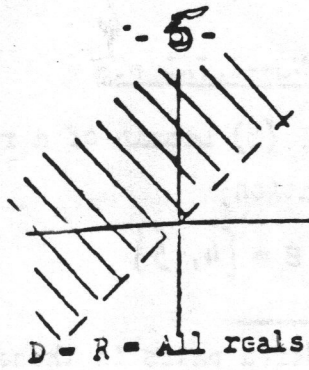
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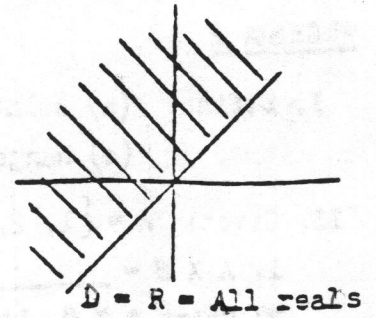
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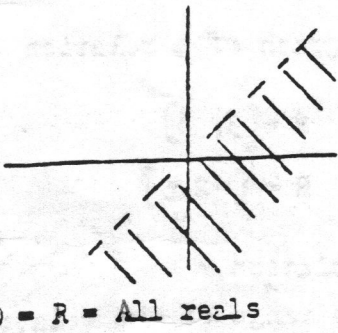
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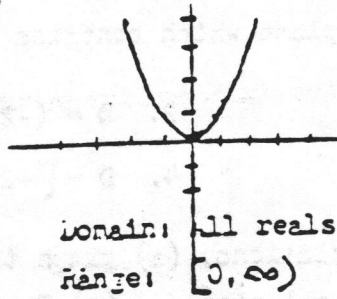
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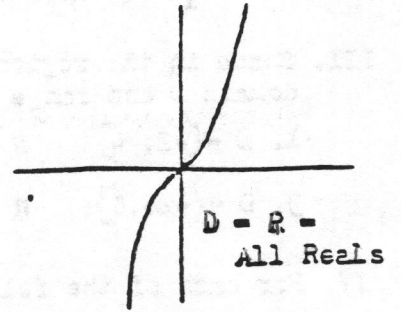
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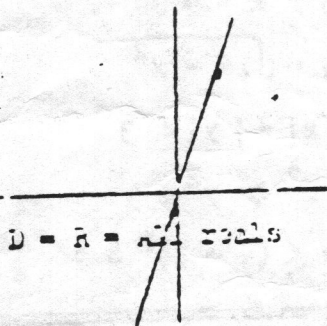
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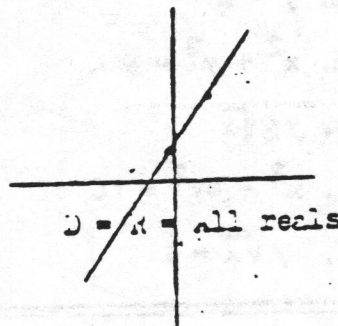
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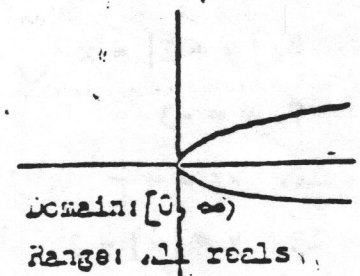
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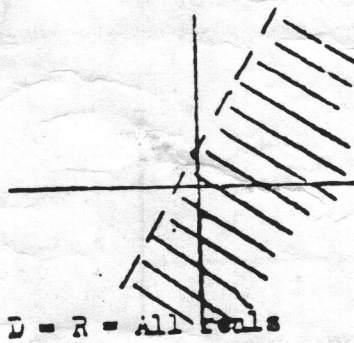
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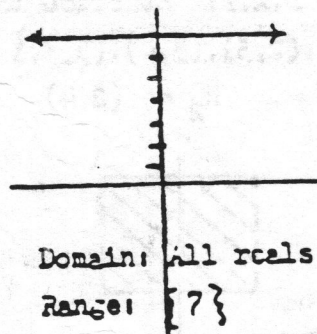
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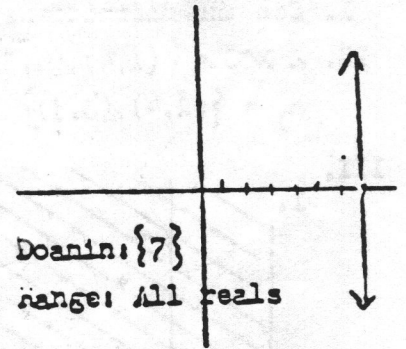
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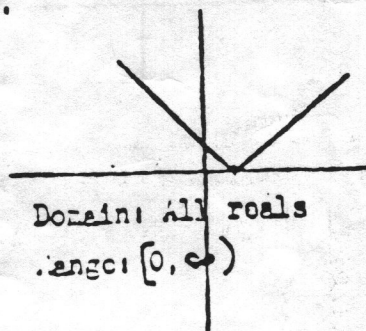
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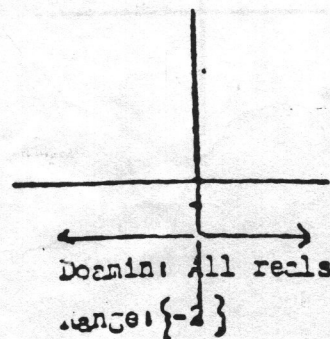
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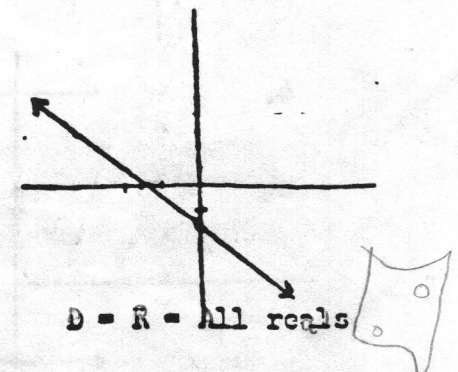
13.



14.



15.



- I. Define: (a) relation; (b) domain of a relation;
(c) range of a relation.

II. Given: $A = \{1, 2, 3\}$ $B = \{4, 5\}$

1. $A \times B =$ _____

2. Using $A \times B$, list the ordered pairs in these relations.

$R_1 = \{(x, y) : y = x + 2\}$

$R_2 = \{(x, y) : y = 2x\}$

III. Shade in the region of the plane which contains the graph of a relation with domain D and range R .

1. $D = [-2, 4]$; $R = [0, \infty)$

2. $D = (-2, 2)$; $R = [3, 6]$

3. $D = (-\infty, 0]$; $R = \{3\}$

4. $D = [-2, 4]$; $R = \{-2, 5\}$

IV. For each of the following relations, (a) graph the relation,

(b) find the domain of the relation,

(c) find the range of the relation.

1. $y = 3x + 2$

2. $y = x^2$

3. $y > x - 4$

4. $|y + 1| = x$

5. $x^2 + y^2 = 9$

6. $y = \sqrt{16 - x^2}$

7. $y = -3$

8. $y \leq \sqrt{x}$

9. $|x| + |y| < 3$

10. $xy = 4$ (hyperbola)

11. $x^2 + 4y^2 = 16$

12. $x^2 \geq y^2$

13. $y = |x| - 3$

14. $y\sqrt{x} = 1$

15. $y \leq (x + 1)^2 - 3$

RELATIONS--TRIAL #11 ANSWERS

I. See the definitions in the L.A.P. Memorize these definitions.

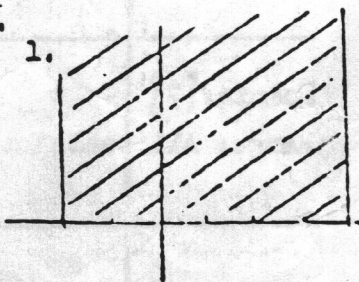
II. $A \times B = \{(1, 4), (1, 5), (2, 4), (2, 5), (3, 4), (3, 5)\}$

$R_1 = \{(2, 4), (3, 5)\}$

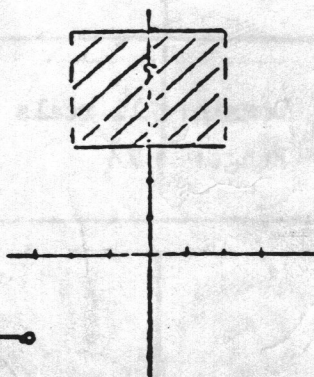
$R_2 = (2, 4)$

III.

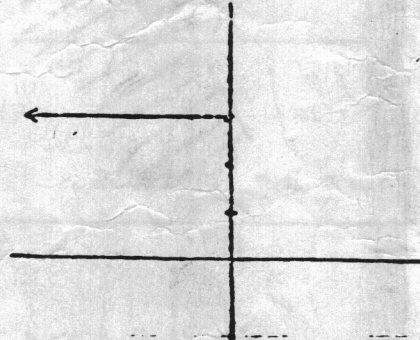
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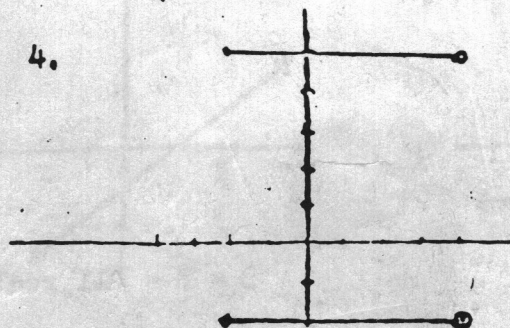
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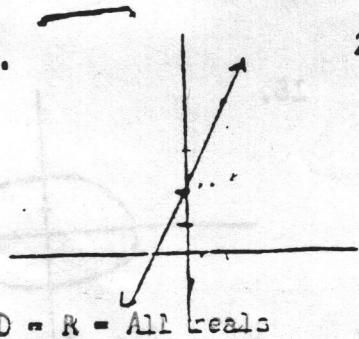
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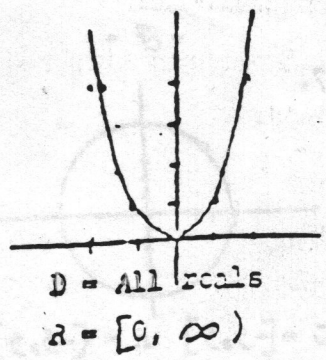
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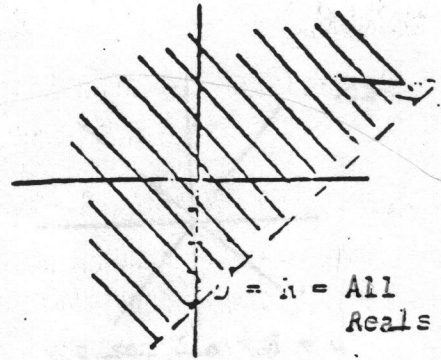
IV. 1.



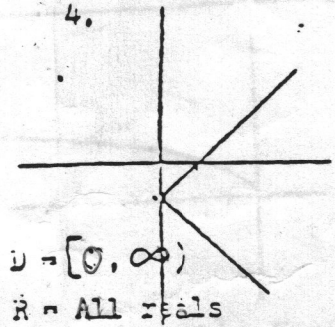
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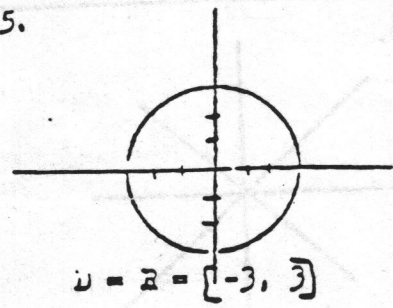
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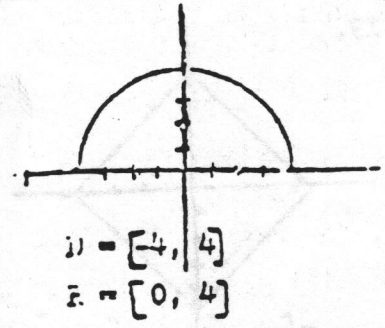
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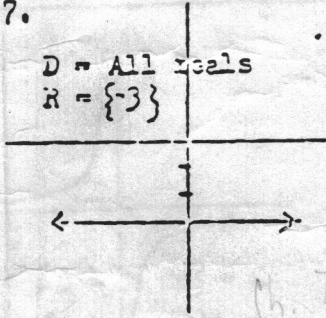
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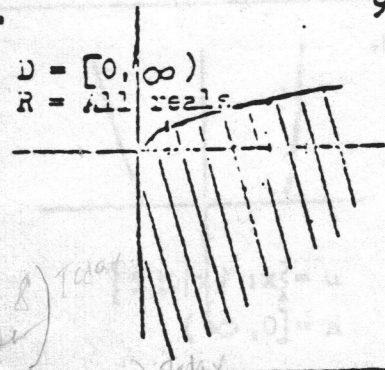
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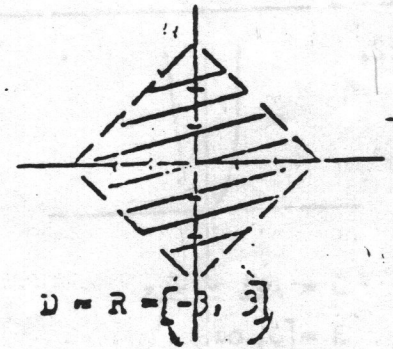
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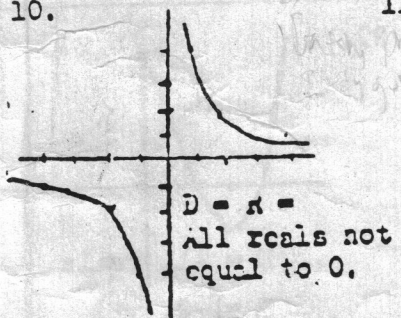
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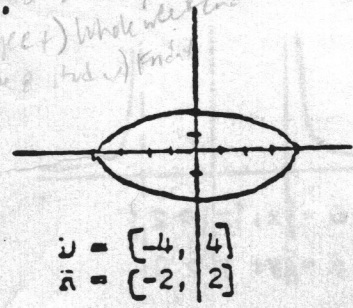
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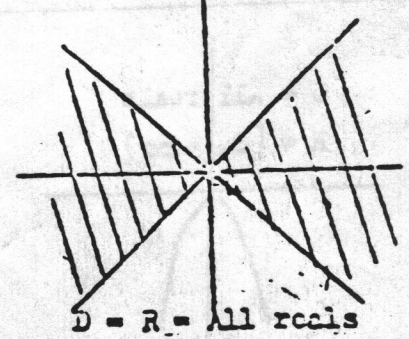
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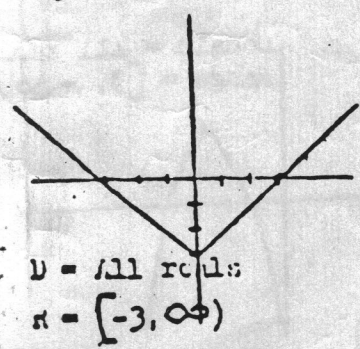
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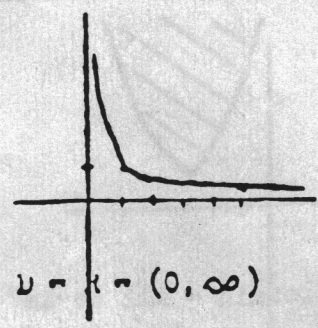
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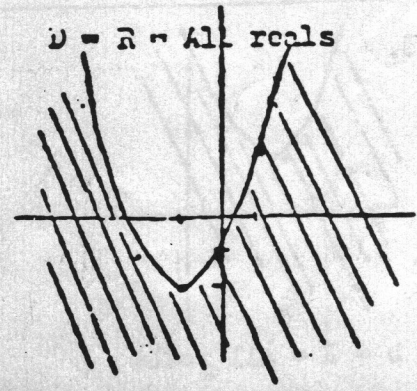
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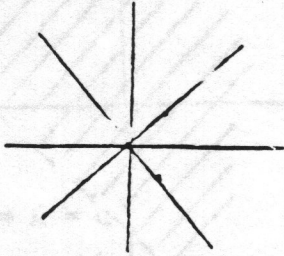
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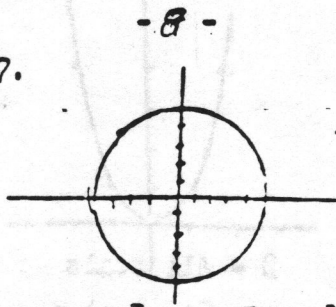


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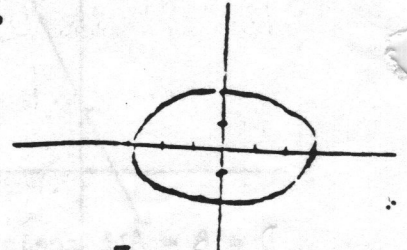
$D = R = \text{All reals}$

17.



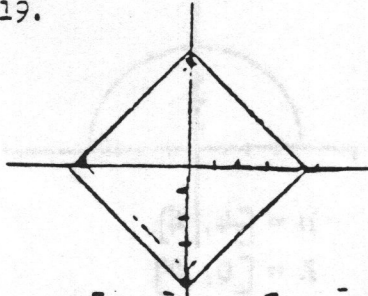
$D = [-5, 5] \quad R = [-5, 5]$

18.



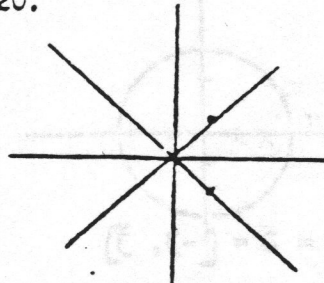
$D = [-3, 3] \quad R = [-2, 2]$

19.



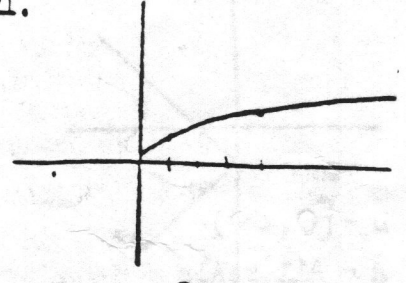
$D = [-4, 4] \quad R = [-4, 4]$

20.



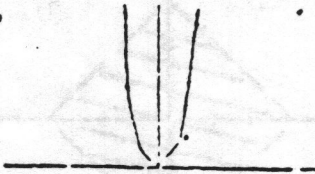
$D = R = \text{All reals}$

21.



$D = R = [0, \infty)$

22.



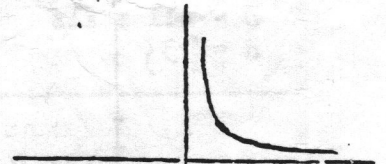
$D = \text{All reals}$
 $R = [0, \infty)$

23.



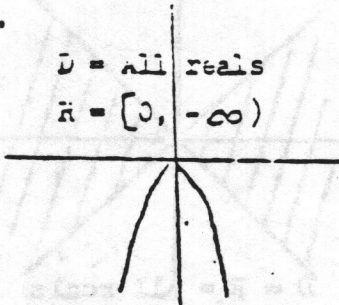
$D = \{x \mid |x| > 2\}$
 $R = [0, \infty)$

24.



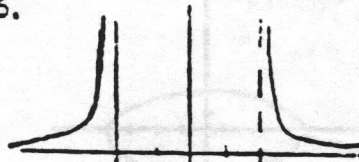
$D = \{x \mid x \neq 0\}$
 $R = \{y \mid y \neq 0\}$

25.



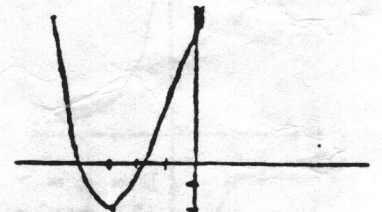
$D = \text{All reals}$
 $R = [0, -\infty)$

26.



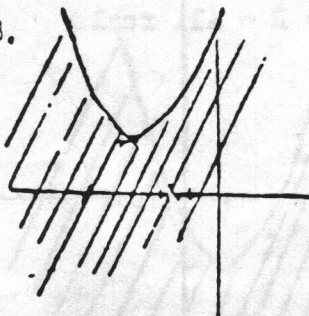
$D = \{x \mid |x| > 2\}$
 $R = \{y \mid y > 0\}$

27.



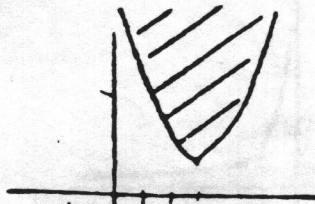
$D = \text{All reals}$
 $R = [-2, \infty)$

28.



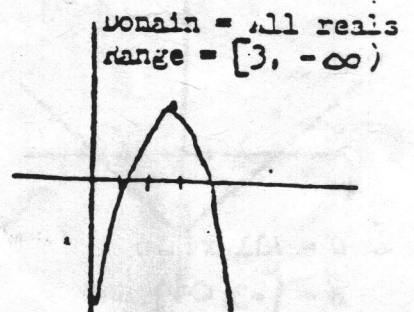
$D = R = \text{All reals}$

29.



Domain = All reals
Range = $[1, \infty)$

30.



Domain = All reals
Range = $[3, -\infty)$