ALGEBRA 2 SEQUENCES AND SERIES

LEARNING ACTIVITIES PACKAGE

BEHAVIORAL OBJECTIVES

I. Identify

A. An arithmetic sequence
B. An arithmetic series
C. A geometric sequence
D. A geometric series

II. Distinguish between

A. A sequence and a series
B. An infinite sequence and a finite sequence
C. An infinite series and a finite series

III. Solve various problems involving sequences and series by employing the following formulas

A. $a_n = a_1 + (n-1)d$

B. $a_n = a_1 r^{n-1}$

C. $S_n = \frac{n(a_1 + a_n)}{2}$ or $S_n = \frac{n(2a_1 + (n-1)d)}{2}$

D. $S_n = \frac{a_1(1 - r^n)}{1 - r}$

E. $S_\infty = \frac{a_1}{1 - r}$

IV. Use $\sum$ (Sigma) notation to express a given series

V. Expand a series expressed in sigma notation

SECTION I ARITHMETIC SEQUENCES

Definition: A sequence is an ordered set of numbers formed according to some pattern.

Exercise 1-A Each of the following is a sequence. Study the terms of the sequence. Discover a pattern and continue the sequence to three more terms.

1. 5, 9, 13, 17, ____, ____, ____
2. 1, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, ____, ____, ____
3. 1, 4, 9, 16, ____, ____, ____
4. 1, 5, 12, 22, 35, ____, ____, ____
5. 1, 8, 27, 64, ____, ____, ____
6. 1, 3, 6, 10, 15, ____, ____, ____
7. 1, 2, 6, 24, 120, ____, ____, ____
8. 1, 1, 2, 3, 5, 8, 13, ____, ____, ____

Definition: An <u>arithmetic</u> sequence is a sequence in which each term after the first is formed by adding a fixed number to the preceding term. The fixed number is called "d", the common difference.

<u>Exercise 1-B</u>

A. Determine the common difference "d", for each of the following arithmetic sequences:

1. 5, 7, 9, 11, . . .
2. 3, 2, 1, 0, . . .
3. 12, 17, 22, 27, . . .
4. 400, 350, 300, . . .

B. Write the first 5 terms for each defined arithmetic sequence below:

1. a_1 (the first term) = 10, and the common difference is 5.
2. $a_1 = 3$, and $d = 6$
3. $a_1 = 12$, and $d = 10$
4. $a_1 = 7$ and $d = -4$
5. $a_1 = 9$, and $d = 0$

In general, an arithmetic sequence is written: $a_1, a_2, a_3, a_4, \ldots, a_n, \ldots$

The subscript identifies the position of the term in the sequence. For instance, a_{20} is the twentieth term.

Given the arithmetic sequence: 4, 7, 10, 13, 16, 19, . . .

Notice: $a_1 = 4$

$a_2 = 4 + 3$

$a_3 = 4 + 2\cdot3$

$a_4 = 4 + 3\cdot3$

$a_5 = 4 + 4\cdot3$

$\vdots$

$a_n = 4 + (n-1)3$

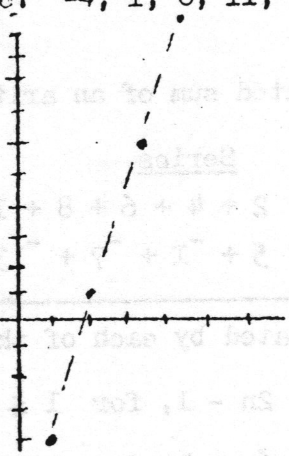
THE PATTERN TO THE LEFT DEMONSTRATES THE GENERAL FORMULA FOR FINDING THE nth TERM OF AN ARITHMETIC SEQUENCE:

$$a_n = a_1 + (n-1)d$$

<u>Exercise 1-C</u> Find the specified term for each of the following sequences:

1. $a_1 = 12$, $d = 2$, $a_{11} =$ ____
2. $a_1 = 3$, $a_2 = 12$, $a_{28} =$ ____
3. $a_1 = 7$, $a_3 = 11$, $a_{101} =$ ____
4. $a_2 = 8$, $a_3 = 4$, $a_{38} =$ ____
5. $a_{11} = 20$, $d = 6$, $a_1 =$ ____

Consider the sequence: $-4, 1, 6, 11, 16, \ldots$



The numbers of an arithmetic sequence, when graphed, as shown in the diagram to the left, lie in a straight line. Notice, the slope of the line is actually the common difference of the sequence. In this case: 5.

When given some initial members of an arithmetic sequence, the general term, a_n can be determined in one of two ways.

1) Thinking in terms of the line: $a_n = 5n + ?$

We know that when $n = 1$, a_n must be -4. Hence: $-4 = 5 + ?$
Consequently, $? = -9$.

Conclusion: $a_n = 5n - 9$

2) Using the formula: $a_n = a_1 + (n - 1)d$

By observing the sequence: $a_1 = -4$, and $d = 5$

Hence: $a_n = -4 + (n - 1)5$

$a_n = -4 + 5n - 5$

$a_n = 5n - 9$

Exercise 1-D Determine a_n for each of the following arithmetic sequences:

1. $6, 12, 18, 24, \ldots$
2. $5, 3, 1, \ldots$
3. $a_1 = 6, \quad d = 5$
4. $a_1 = 17, \quad d = 2$
5. $22, 32, 42, \ldots$
6. $107, 100, 93, \ldots$
7. $a_1 = 4, \quad a_2 = 70$
8. $a_1 = 69, \quad a_2 = 50$
9. $a_2 = 6, \quad d = 3$
10. $a_4 = 5, \quad d = -3$

SECTION II

ARITHMETIC SERIES

DEFINITION: An arithmetic series is the indicated sum of an arithmetic sequence.

Sequence	Series
2, 4, 6, 8, 10	$2 + 4 + 6 + 8 + 10$
5, -1, -7, -13	$5 + {}^{-}1 + {}^{-}7 + {}^{-}13$

Exercise 2-A Write the arithmetic series indicated by each of the following:

1. $a_1 = 2$, $d = 4$, to 6 terms
2. $a_n = 2n - 1$, for $1 \leq n \leq 8$
3. $a_2 = 6$, $d = 3$, for 4 terms
4. $a_n = 6n + 1$, for $1 \leq n \leq 10$

Mathematicians often use the capital Greek letter (sigma) to write a series concisely. The Greek sigma corresponds to the English "S", which is the first letter of the word sum.

$$\sum_{n=1}^{5} (4n + 1)$$

5 ← (upper index)

n = 1 ← (lower index)

To expnad this sigma expression, generate each term by letting n = 1, then 2, etc., incrementing by one, until finally n = 5.

Examples:

1) $\sum_{n=1}^{4} 2n = 2 + 4 + 6 + 8$

2) $\sum_{n=1}^{5} (n + 3) = 4 + 5 + 6 + 7 + 8$

If given an arithmetic series such as: $7 + 12 + 17 + 22 + 27 + 32 + 37$.
To express the series in sigma notation, proceed as follows:

1) Determine the general term: a_n

Note, for the above: $a_n = 5n + 2$ ← The value needed to obtain 7 when n = 1.

(5: The common difference)

2) Determine the number of terms needed. In this case it is seven.

3) Check out for n=7. Note: $a_7 = 5 \cdot 7 + 2 = 37$. That is good!

Hence, the $\sum$ notation is: $\sum_{n=1}^{7} (5n + 2)$

Exercise 2-B

1. Expand each of the following

A. $\sum_{n=1}^{3} (7n - 2)$

B. $\sum_{n=1}^{12} (4n - 3)$

C. $\sum_{n=1}^{6} 2n$

D. $\sum_{n=1}^{2} (5n + 1)$ E. $\sum_{n=1}^{1} (62n - 3)$ F. $\sum_{n=1}^{4} (n - \frac{1}{2})$

2. Write each of the following arithmetic series using $\sum$ notation.

A. $3 + 4 + 5 + 6 + 7 + 8 + 9 + 10$

B. $62 + 52 + \ldots + 12$

C. $0 + 4 + 8 + 12$

D. $5 - 4 - 13$

E. $1 + 2 + 3 + \ldots + 100$

F. $8 + 12 + 16 + \ldots + 56$

Sometimes it is necessary to actually find the sum of an arithmetic series. A neat thing happens if we play a little trick!

Suppose we wish to add the integers from 1 to 100 inclusive.

1) Write the series: $1 + 2 + 3 + 4 + \ldots + 99 + 100$

2) The series backwards: $100 + 99 + 98 + 97 + \ldots + 2 + 1$

3) Add: $101 + 101 + 101 + \ldots + 101 + 101$

4) Now, we have 101, 100 times! BUT, WE ALSO HAVE THE SUM OF THE SERIES TWICE.

5) So, the sum is $\frac{100 \times 101}{2} = 5050$

6) THE SUM OF THE FIRST 100 POSITIVE INTEGERS IS 5050.

IN GENERAL FOR AN ARITHMETIC SERIES SUM:

$$S_n = a_1 + (a_1 + d) + a_1 + 2d) + \ldots + (a_1 + (n-1)d$$

$$S_n = a_1 + (n-1)d + (a_1 + (n-2)d) + (a_1 + (n-3)d) + \ldots + a_1$$

ADD: $$2S_n = (2a_1 + (n-1)d) + (2a_1 + (n-1)d) + (a_1 + (n-1)d) + \ldots + (a_1 + (n-1)d)$$

$$2S_n = n(a_1 + (n-1)d)$$

(1) $$\boxed{S_n = \frac{n(2a_1 + (n-1)d)}{2}}$$ (S_n is read S sub n.)

Since $a_n = a_1 + (n - 1)d$, the above formula can also be expressed as:

(2) $$\boxed{S_n = \frac{n(a_1 + a_n)}{2}}$$

Formula (1) is helpful when given the first term, the common difference and the number of terms.

(2) is helpful when given the first term, the last term and the nubmer of terms.

Exercise 2-C Determine S_n for each of the following:

1. $a_1 = 12, a_n = 42, n = 12$
2. $a_1 = 7, d = 5, n = 50$
3. $\sum_{n=1}^{20} (2n - 3)$
4. $\sum_{n=1}^{100} (3n + 2)$
5. $4 + 8 + 12 + 16 + \ldots + 44$
6. $a_n = 6n - 3, n = 40$
7. $a_n = -2n + 1, \quad n = 70$
8. $a_1 = 6, d = 30, n = 61$
9. $\sum_{n=1}^{3} (400m - 2)$
10. $\sum_{n=1}^{80} (n + 20)$
11. $a_1 = 300, d = 6, n = 41$
12. $a_1 = 2, n = 60, d = 4$
13. $a_1 = 24, a_2 = 30, n = 31$
14. $a_2 = 16, a_3 = 20, n = 48$

SECTION III GEOMETRIC SEQUENCES

DEFINITION: A geometric sequence is a sequence in which each term after the first is formed by multiplying the preceding term by a fixed number. The fixed number is called the common ratio.

Exercise 3-A Determine the common ratio, "r" for each of the following geometric sequences:

1. $3, 6, 12, 24, \ldots$
2. $4, \frac{4}{3}, \frac{4}{9}, \frac{4}{27}, \ldots$
3. $4, 2, 1, \frac{1}{2}, \frac{1}{4}, \ldots$
4. $2, -2, 2, -2, 2, \ldots$
5. $4, 4, 4, 4, \ldots$
6. $1, -3, 9, -27, \ldots$
7. $3, 2, \frac{4}{3}, \frac{8}{9}, \ldots$
8. $\log 3, \log 9, \log 81, \ldots$

The general geometric sequence is written:

$$a_1, a_1r, a_1r^2, a_1r^3, a_1r^4, \ldots, a_1r^{n-1},$$

The general term of a geometric sequence is: $\boxed{a_n = a_1r^{n-1}}$

Exercise 3-B

1. For each of the following, find the required term:

A. $a_1 = 7, \quad r = 2, \quad a_4 = ?$

B. $a_1 = 3, a_2 = 6, \quad a_{12} = ?$

C. $a_2 = 12, a_3 = 3, \quad a_{17} = ?$

D. $a_1 = 7, r = .1, \quad a_{12} = ?$

E. $a_1 = .3$, $r = .1$, $a_{15} =$ F. $a_1 = t$, $r = p$, $a_{20} =$

II. Write the first 5 terms of each geometric sequence defined below:

A. $a_1 = 7$, $r = 2$; B. $a_1 = 3$, $a_2 = 6$; C. $a_1 = -4$, $a_2 = 2$;

D. $a_1 = 4$, $a_2 = 2$; E. $a_1 = .3$, $r = 10$

III. Determine a_n for each geometric sequence defined below:

A. $a_1 = 9$, $r = 3$; B. $a_2 = 12$, $r = \frac{1}{3}$; C. $a_1 = 10$, $a_2 = 1$;

D. $a_1 = x$, $r = y$; E. $a_4 = 8$, $a_3 = 4$; F. $a_5 = 243$, $a_2 = 9$

SECTION IV. GEOMETRIC SERIES

Definition: A geometric series is the indicated sum of a geometric sequence.

Examples:

Sequence	Series
2, 4, 8, 16, 32	2 + 4 + 8 + 16 + 32
$1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}$	$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8}$
3, .3, .03, .003, .0003	3 + .3 + .03 + .003 + .0003
a, ar, ar^2, ar^3, ar^4	$a + ar + ar^2 + ar^3 + ar^4$

In general, a geometric series can be written in the form:

$$\sum_{n=1}^{k} a\,r^{n-1}$$ where n and k are members of the set of positive integers.

Again, it is relatively simple to discover a formula for finding the sum of a given series.

Sample:

$$S_n = 1 + 3 + 9 + 27 + 81$$
$$3S_n = 3 + 9 + 27 + 81 + 243$$
(Subtract--top from bottom.)
$$2S_n = -1 + 243$$
$$S_n = 121$$

To generalize:

$$(1) \quad S_n = a_1 + a_1r + a_1r^2 + a_1r^3 + \ldots + a_1r^{n-1}$$

Multiply by r: $$(2) \quad rS_n = \quad a_1r + a_1r^2 + a_1r^3 + \ldots + a_1r^{n-1} + a_1r^n$$

Subtract: $S_n - rS_n = a_1 - a_1r^n$

Factor: $S_n(1 - r) = a_1(1 - r^n)$

Divide: $$\boxed{S_n = \frac{a_1(1 - r^n)}{1 - r}}$$

The above is the formula for finding the sum of a geometric series of n terms. The formula fails for $r = 1$. However, for $r = 1$, the series can be expanden for a few terms and the method of summing can be easily determined. The management will not deny you the pleasure of overcoming this obstacle.

Exercise 4-A

Find the sum of each geometric series defined below: (Leave answers in exponential form.)

1. $a_1 = 4$, $r = \frac{1}{2}$, $n = 12$; B. $a_1 = 6$, $a_2 = 24$, $n = 6$

C. $a_1 = 2$, $r = 1$, $n = 500$; D. $a_2 = 12$, $a_3 = 9$, $n = 7$

E. $a_2 = 7$, $a_3 = \frac{2}{7}$, $n = 20$

It is hard to believe, but some infinite geometric series; i.e. a series containing an infinite number of terms, have finite sums.

Study the following expensions for 1:

$$1 = \frac{1}{2} + \frac{1}{2}$$

$$1 = \frac{1}{2} + \frac{1}{4} + \frac{1}{4}$$

$$1 = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{8}$$

$$1 = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{16}$$

$$1 = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \frac{1}{32}$$

Really, couldn't we go on in this fashion forever?

We claim $\sum_{n=1}^{\infty} (\frac{1}{2})^n = 1$ (The symbol ∞ means infinity.)

Exercise 4-B

1. Evaluate each of the following:

A. $(\frac{1}{10})^2$ B. $(\frac{1}{10})^4$ C. $(\frac{1}{2})^8$ D. $(\frac{1}{3})^4$

E. $(\frac{1}{10})^{10}$ F. $(\frac{1}{10})^n$

2. Arrange the following numbers in descending order:

$(\frac{1}{2})^6$; $(-\frac{1}{2})^5$; $(\frac{1}{2})^3$; $(\frac{1}{2})^{12}$; $(-\frac{1}{2})^7$; $(\frac{1}{2})^{100}$; $(-\frac{1}{2})^{80}$; $(\frac{1}{2})^0$; $(\frac{1}{2})^{16}$

3. In general for $|r| < 1$, as n gets very large, r^n tends to ________.

When the ratio, r, of a geometric series is such that $|r| < 1$, the infinite series has a sum because as n tends to infinity, r^n tends to 0.

Thus the formula for the sum of a geometric series which is $S_n = \frac{a_1(1 - r^n)}{1 - r}$

becomes simply: $S = \frac{a_1}{1 - r}$

BEWARE: ONLY INFINITE GEOMETRIC SERIES WITH $|r| < 1$ HAVE A FINITE SUM.

Exercise 4-C

Find the sum of each of the following infinite geometric series:

1. $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \ldots$
2. $a_1 = 7,\ r = \frac{2}{3}$
3. $r = -\frac{1}{3},\ a_1 = 9$
4. $1 + \frac{1}{2} + \frac{1}{4} + \ldots$
5. $6 + \frac{12}{5} + \frac{24}{25} + \ldots$
6. $a_2 = 3,\ r = \frac{1}{8}$
7. $a_2 = 7,\ r = \frac{2}{3}$
8. $a_3 = 7,\ r = \frac{2}{3}$
9. $a_1 = 3,\ r = 3$
10. $1 + \frac{1}{3} + \frac{1}{9} + \ldots$
11. $a_1 = 10,\ r = .1$
12. $a_1 = .3,\ r = .1$

COMPLETE KNOWLEDGE OF THE L.A.P. THUS FAR IS SUFFICIENT FOR THE COURSE. TEACHERS HAVE A WAY OF GETTING CARRIED AWAY. SHOULD YOU BE OF A CREATIVE NATURE AND ENJOY A MATHEMATICAL CHALLENGE TRY SOME OR ALL OF THE PROBLEMS ON THE NEXT PAGE. THE MANAGEMENT PUT THEM IN FOR SHEER DELIGHT AND YOUR ENTERTAINMENT.

SECTION V

A CREATIVE VENTURE

Solve each of the following problems. Try them out on some Math Analysis Students just for fun.

1. Solve for x: $\frac{1+x}{x} = 1 + x + x^2 + \ldots$

2. What distance will a dead golf ball travel if it is dropped from a height of 70 inches and if after each fall it rebounds $\frac{1}{10}$ of the distance it fell?

3. Solve for x: $\frac{2}{3} = 1 + x + x^2 + \ldots$

4. Evaluate: $\sum_{k=1}^{4} (2^k - k)$

5. 20 Pet Rocks are placed on the ground at intervals of 5 yards apart. A runner has to start from a basket 5 yards from the first Pet Rock, pick up the Pet Rocks and bring them back to the basket one at a time. How many yards has he to run altogether?

6. Find the sum of the even integers from 10 to 58 inclusive.

7. Find x if $(3 - x)$, $-x$, $9 - 2x$ are in atithmetic sequence.

8. Find the sum of all positive integers less than 300 which are multiples of 7..

9. Find n if $\sum_{k=0}^{n} 2^k = 63$

10. Find x so that the sequence is geometric: $-\frac{3}{2}, x, -\frac{8}{27}$.

ANSWERS

1-A. 1. 21, 25, 29 2. $\frac{1}{5}, \frac{1}{6}, \frac{1}{7}$ 3. 25, 36, 49

4. 51, 70, 92 5. 125, 216, 343 6. 21, 28, 36

7. 6!, 7!, 8! 8. 21, 34, 55

1-B. A. 1. 2; 2. -1, 3. 5; 4. -50

B. 10, 15, 20, 25, 30; 2. 3, 9, 15, 21, 27; 3. 12, 22, 32, 42, 52

4. 7, 3, -1, -5, -9; 5. 9, 9, 9, 9, 9

1-C 1. 32; 2. 246; 3. 207; 4. -136; 5. -40

ANSWERS CONTINUED

Exercises 1-D   1. $6n$;   2. $-2n + 7$;   3. $5n + 1$;   4. $2n + 15$
5. $10n + 12$ 6. $-7n + 114$;   7. $66n - 62$;   8. $-19n + 88$
9. $3n$;   10. $-3n + 17$

2-A   1. $2 + 6 + 10 + 14 + 18 + 22$;   2. $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15$
3. $3 + 6 + 9 + 12$ 4. $7 + 13 + 19 + 25 + 31 + 37 + 43 + 49 + 55 + 61$

2-B   1. A. $5 + 12 + 19$;   B. $1 + 5 + 9 + 13 + 17 + 21 + 25 + 29 + 33 + 37 + 41 + 45$
C. $2 + 4 + 6 + 8 + 10 + 12$ D. $6 + 11$ E. 59   F. $1/2 + 3/2 + 5/2 + 7/2$

2. A. $\sum_{n=1}^{8} (n + 2)$;   B. $\sum_{n=1}^{6} (-10n + 72)$ C. $\sum_{n=1}^{4} (4n - 4)$

D. $\sum_{n=1}^{3} (-9n + 14)$ E. $\sum_{n=1}^{100} n$ F. $\sum_{n=1}^{13} (4n + 4)$

2-C   1. 324;   2. 6475;   3. 360;   4. 15,350;   5. 264;
6. 4800;   7. -4900;   8. 55,266;   9. 2394;   10. 4840;
11. 17,220;   12. 7,200;   13. 3,534;   14. 5,088

3-A.   1. 2;   2. $-\frac{1}{3}$;   3. $\frac{1}{2}$;   4. -1;   5. 1;   6. -3;
7. 2/3;   8. 2

3-B.   1. 56 or $7 \cdot 2^3$;   B. $3 \cdot 2^{11}$;   C. $48 \cdot (\frac{1}{4})^{16}$;   D. $7 \cdot (.1)^{11}$
E. $.3 \cdot (.1)^{14}$;   F. $t \cdot p^{19}$

II. A. 7, 14, 28, 56, 112;   B. 3, 6, 12, 24, 48;
C. -4, 2, -1, $\frac{1}{2}$, $-\frac{1}{4}$ D. 4, 2, 1, $\frac{1}{2}$, $\frac{1}{4}$
E. .3, 3, 30, 300, 3,000

III. A. $9 \cdot (3)^{n-1}$;   B. $36 \cdot (\frac{1}{3})^{n-1}$;   C. $10 \cdot (.1)^{n-1}$;
D. $x \cdot y^{n-1}$;   E. 2^{n-1};   F. 3^n

4 A.
a. $8-2^{-9}$ or $8(1-1/2^{12})$
b. $-2+2^{13}$ or $-2(1-4^{6})$
c. 1,022
d. $S_7 = 64[1-(3/4)^7]$
e. $S_{20} = \frac{343}{2}[1-(2/49)^{20}]$

4-B 1. A. .01; B. .0001; C. .00000001; D. $\frac{1}{81}$;
E. .0000000001; F. . (n-1 zeros)1

2. $(\frac{1}{2})^0$, $(\frac{1}{2})^3$, $(\frac{1}{2})^6$, $(\frac{1}{2})^{12}$, $(\frac{1}{2})^{16}$, $(-\frac{1}{2})^{80}$, $(\frac{1}{2})^{100}$, $(-\frac{1}{2})^7$, $(-\frac{1}{2})^5$

3. 0

4-C 1. 2/3; 2. 21; 3. 27/4; 4. 2; 5. 10; 6. 192/7;
7. 63/2; 8. 189/4; 9. Gotcha! the ratio is 3 and 3 > 1.
10. 3/2; 11. $11.\overline{1}$ 12. $.\overline{3}$

Section V

1. $x = \frac{-1 \pm \sqrt{5}}{2}$ 2. 85.5 inches; 3. $-\frac{1}{2}$, 4. 20

5. 2,100 6. 850; 7. 12; 8. 6321;

9. 5; 10. $\pm$ 4/9

ALGEBRA 2

SEQUENCES AND SERIES
TRIAL RUN

I. Matching. Match the description with the common notation.

1. Common difference	A. S_∞
2. Common ratio	B. S_n
3. Number of terms	C. r
4. Sum of n terms	D. a_n
5. First term	E. a_1
6. Sum of an infinite number of terms	F. n
7. nth term	8. d

II. For each of the following, determine the indicated information. Each of the following is either an arithmetic or a geometric sequence or series.

1. 6, 4, 2, 0, . . .	(a) a_{20}	(b) d	(c) a_n
2. 3, .3, .03, .003, . . .	(a) r	(b) a_n	
3. 1, 3, 5, 7, . . .	(a) d	(b) a_1	(c) a_n
4. 10, 20, 30, . . .	(a) a_{30}	(b) d	(c) a_n
5. 15, 10, 5, 0, . . .	(a) a_{30}	(b) d	(c) a_n
6. $1 + \frac{2}{3} + \frac{4}{9} + \cdots$	(a) r	(b) S_∞	(c) a_n
7. $1 - \frac{2}{3} + \frac{4}{9} - \cdots$	(a) r	(b) S_{10}	(c) a_n

III. For each of the following determine a pattern and continue the sequence for three more terms.

1. 6, 4, 3, 3, 4, ___, ___, ___
2. 4, 2, 1, $\frac{1}{2}$, ___, ___, ___
3. 5, 7, 4, 6, 3, ___, ___, ___
4. 3, 3, 6, 9, 15, ___, ___, ___
5. 1.3, 1, 2.3, 2, ___, ___, ___
6. 1, 2, 5, 10, 17, ___, ___, ___

IV. Expand each of the following:

1. $\sum_{n=1}^{6} (3n + 2)$
2. $\sum_{n=1}^{5} \left(\frac{2 + n}{n}\right)$
3. $\sum_{n=1}^{4} (2n + 1)$
4. $\sum_{n=1}^{4} 2n + 1$

V. Write each of the following using sigma notation:

1. $2 + 4 + 6 + 8$
2. $6 + 9 + 12 + \ldots + 33$
3. $.6 + .06 + .006 + \ldots$
4. $1 + \frac{1}{2} + \frac{1}{4} + \ldots$
5. $12 + 5 - 2 - 9 - \ldots - 135$

VI. Match each of the following with the proper formula

1. The sum of n terms of an arithmetic series
2. The nth term of an arithmetic sequence
3. The sum of n terms of a geometric series
4. The nth term of a geometric sequence
5. The sum of an infinite number of terms of a geometric series

A. $a_1 + (n - 1)d$

B. $\dfrac{n(a_1 + a_n)}{2}$

C. $\dfrac{a_1}{1 - r}$

D. $\dfrac{n(2a_1 + (n-1)d)}{2}$

E. $\dfrac{a_1(1 - r^n)}{1 - r}$

F. $a_1 r^{n-1}$

VII. Solve each of the following:

1. If the third term of an arithmetic sequence is -20 and the 35th term is 28, what is the first term?
2. If the first term of a geometric sequence is $\frac{5}{9}$ and the fifth term is 45, what is the common ratio?
3. If a_1 is -35 and a_3 is -5, find a_{15} if the sequence is arithmetic.
4. Find the sum of all positive integers which are less than 200 and divisible by 7.
5. If the sum of an arithmetic series of n terms is 500, $a_1 = 2$, and $a_n = 48$, then n is ______.
6. Determine the sum of the even integers between 3 and 501.
7. Find x so that $2x + 1$, $4x$, and $5x + 1$ form an arithmetic sequence.
8. Find a_1 if $a_n = 41$, $n = 18$, and $d = 2$.
9. The sum of an infinite geometric series is 6, and $a_1 = 3$. What is the value of r?

SEQUENCES AND SERIES TRIAL RUN ANSWERS

I. 1. G 2. C 3. F 4. B 5. E 6. A 7. D

II. 1. (a) -32 (b) -2 (c) -2n + 8 2. (a) .1 (b) $3(.1)^{n-1}$

3. (a) 2 (b) 1 (c) 2n - 1 4. (a) 300 (b) 10 (c) 10n

5. (a) -130 (b) -5 (c) -5n + 20 6. (a) 2/3 (b) 3 (c) $(2/3)^{n-1}$

7. (a) -2/3 (b) $\frac{3}{5}\left[1-\left(-\frac{2}{3}\right)^{10}\right]$ (c) $\left(-\frac{2}{3}\right)^{n-1}$

III. 1. 6, 9, 13 2. 1/4, 1/8, 1/16 3. 5, 2,,4

4. 24, 39, 63 5. 3.3, 3, 4.3 6. 26, 37, 50

IV. 1. 5 + 8 + 11 + 14 + 17 + 20 2. 3 + 2 + 5/3 + 3/2 + 7/5

3. 3 + 5 + 7 + 9 4. 2 + 4 + 6 + 8 + 1

V. $\sum_{n=1}^{4} 2n$ 2. $\sum_{n=1}^{10} (3n + 3)$ 3. $\sum_{n=1}^{\infty} 6(.1)^{n}$

4. $\sum_{n=1}^{\infty} (1/2)^{n-1}$ 5. $\sum_{n=1}^{22} (-7n + 19)$

VI. 1. B and D 2. A 3. E 4. F 5. C

VII. 1. -23 2. 3 3. 175 4. 2842

5. 20 6. 62,748 7. 2 8. 7

9. 1/2