SPECIAL FUNCTIONS

For each function defined in this L.A.P.

(a) Graph the function
(b) Determine the domain of the function
(c) Determine the range of the function
(d) Describe the symmetry of the graph of the function
(e) Determine whether the function is odd, even, or neither.
(f) Describe the monotonicity of the function over the domain

1. $(-\infty, \infty)$; 2. $(-\infty, 0]$; 3. $[0, \infty)$

(g) Graph the converse of the function
(h) Determine whether or not the function has an inverse.

1. The Constant Function: $f(x) = k$ [k is some real number]

   For graphing purposes let k = 3.

2. The Multiplying Function: $f(x) = kx$ [k is some real number]

   Graph three cases of this function; i.e. let k = 3; -3; 0.
   Answer (f) relative to k positive, zero, negative.
   What ordered pair is common to all multiplying functions?
   Answer (h) carefully -- check all three cases.

3. The Reciprocating Function: $f(x) = \frac{1}{x}$

4. The Squaring Function: $f(x) = x^2$

5. The Cubing Function: $f(x) = x^3$

6. The Absolute Value Function: $f(x) = |x|$

7. The Greatest Integer Function: $f(x) = [\![x]\!]$

   Here's a new function for you. It is read f(x) = the greatest integer which does not exceed x. It is commonly known as the step function because the graph is a bunch of horizontal half-open segments. Study these samples: $f(2) = 2$; $f(3.2) = 3$; $f(3.9) = 3$; $f(-2.8) = -3$

8. The Fractional Part Function: $f(x) = \{\{x\}\}$

   Another new one. $\{\{x\}\} = x - [\![x]\!]$. This could be called the slash function because the graph is a bunch of diagonal half-open segments. Another cute thing about this function is that it is periodic. Like a broken record, it keeps telling the same story over and over. The period of the function is one.
   Study these samples: $f(2.3) = .3$; $f(7.9) = .9$; $f(-4.2) = .8$;

   $f(3) = 0$

9. The Exponential Function: $f(x) = k^x$ $[k \geq 0]$

Consider four cases for this function; i.e.

$k > 1$; $k = 1$; $0 < k < 1$; $k = 0$.

What ordered pair do all exponential functions with $k > 0$ have in common?

10. The Log Function: $f(x) \log_k x$ $[k > 0 \text{ and } k \neq 1]$.

Graph at least three cases of this function; i.e.

$k = 2$; $k = 3$; $k = \frac{1}{2}$

Do you remember $y = \log_k x \iff k^y = x$

What ordered pair do all log functions have in common?

11. The Signum Function: $f(x) = \text{sgn } x$

The signum function is defined thus:

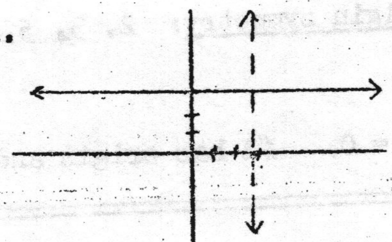
$$\text{syn } x = \begin{cases} 1 \text{ for } x z 0 \\ 0 \text{ for } x = 0 \\ -1 \text{ for } x < 0 \end{cases}$$

Because of the appearance of the graph we might call this the old slide-point-slide function.

When you have become an authority on the preceding eleven functions take a trial run. Check over any trouble spots and then on to the real thing.

(a) Graphs: and (g) converses. (The converses are sketched with dashed lines.)

1.



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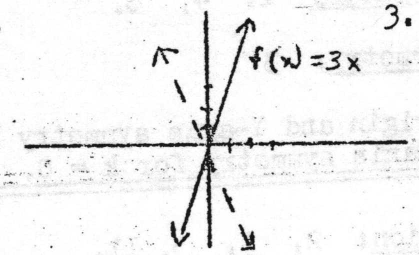

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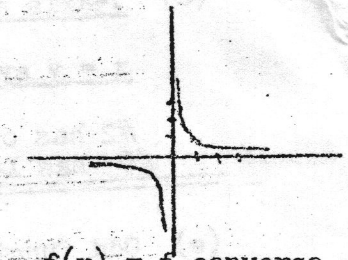
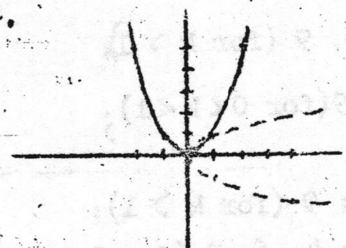


f(x) = f converse

4.



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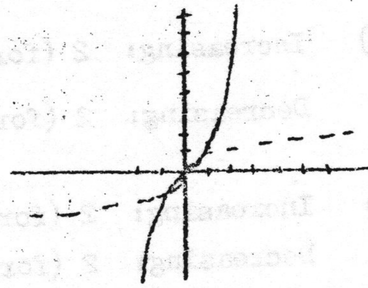

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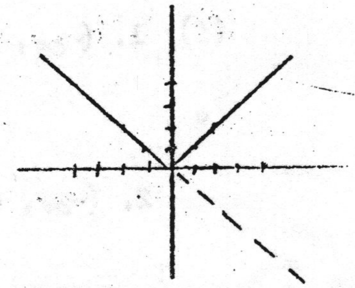

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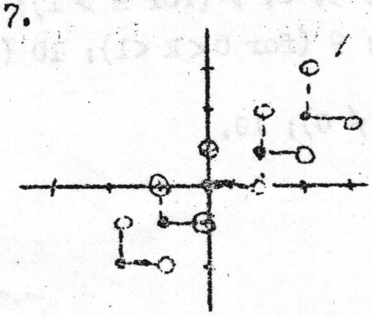

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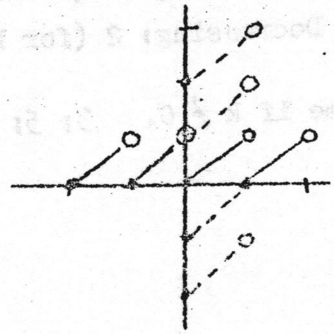
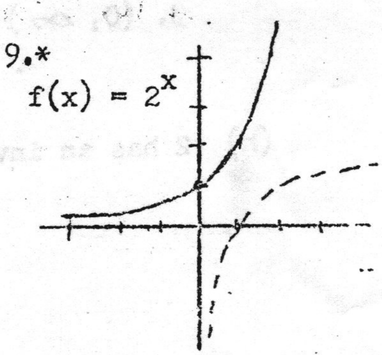


9.*



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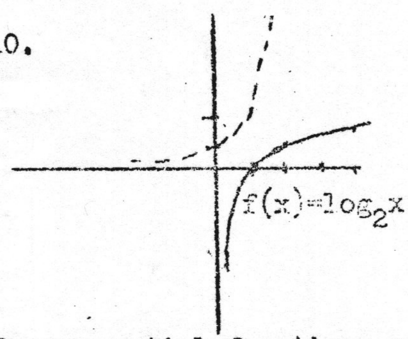

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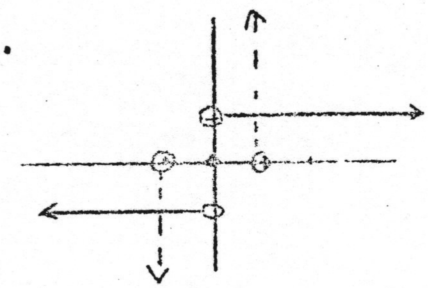

* All exponential functions with k 0 have the ordered pair (0,1) in common. When k = 1 the function is constant. When k = 0 the function's graph is on the positive X-axis.

(b) <u>DOMAINS</u>: <u>All reals</u>: 1, 2, 4, 5, 6, 7, 8, 9, 11. 3.: $x \neq 0$. 10. $x > 0$.

(c) <u>RANGES</u>: <u>All reals</u>: 2, 5, 9. $\{y: y \geq 0\}$: 4, 6, $\{y: y = k\}$: 1.

$\{y: y \neq 0\}$: 3. Integers: 7 $\{y: 0 \leq y < 1\}$: 8.

$\{-1, 0, 1\}$: 11.

For #9: $k > 1$: $\{y: y > 0\}$; $k = 1$: $\{y: y = k\}$, $0 < k < 1$; $\{y: y > 0\}$.

$k = 0$: $\{1\}$

(d) Y-axis symmetry: 1, 4, 6. Origin symmetry: 2, 3, 5, 11.

x = y symmetry: 3

#2 has origin and Y-axis symmetry for k = 0. #1 has origin and Y-axis for k=
#9 has X-axis symmetry for k = 0.

(e) Odd function: 2, 3, 5, 11. Even function: 1, 4, 6

(f) 1. $(-\infty, \infty)$ Increasing: 2 (for $k > 0$; 5, 9 (for $k > 1$);

Decreasing: 2 (for $k < 0$), 9(for $0 < k < 1$);

2. $(-\infty, 0)$: Increasing: 2 (for $k > 0$); 5; 9 (for $k > 1$);

Decreasing: 2 (for $k < 0$); 3; 4; 6; 9 (for $0 < k < 1$)

3. $(0, \infty)$: Increasing: 2 (for $k > 0$); 4; 5; 6; 9 (for $k > 1$); 10 (for $k > 1$)

Decreasing: 2 (for $k < 0$); 3; 9 (for $0 < k < 1$); 10 (for $0 < k < 1$)

(h) 2 has an inverse if $k \neq 0$. 3; 5; 9 (for $k \neq 0$); 10.

ALGEBRA 2 SPECIAL FUNCTIONS TRIAL RUN

I. Consider the functions:

A. $f(x) = 2$ B. $f(x) = 3x$ C. $f(x) = -6x$ D. $f(x) = \log_2 x$

E. $f(x) = [\![x]\!]$ F. $f(x) = \text{sgn } x$ G. $f(x) = 2^{-x}$ H. $f(x) = \frac{1}{x}$

(The management suggests that you graph each of the above functions...straight away.)

Which of the above functions

(a) are strictly increasing over the domain $(0, \infty)$?

(b) have a domain $(-\infty, \infty)$?

(c) have a range $\{-1,0,1\}$?

(d) have inverses?

(e) have Y-intercepts zero?

(f) have a range $(-\infty, \infty)$?

(g) have a range $(0, \infty)$?

(h) are strictly decreasing over the domain $(0, \infty)$?

II. Give the domain and range for each of the following functions:

FUNCTION	DOMAIN	RANGE
(a) $f(x) = [\![x]\!]$	______	______
(b) $f(x) = 2^x$	______	______
(c) $f(x) = \lvert x \rvert$	______	______
(d) $f(x) = \text{sgn } x$	______	______
(e) $f(x) = \log_3 x$	______	______
(f) $f(x) = \{\!\{x\}\!\}$	______	______
(g) $f(x) = x^3$	______	______
(h) $f(x) = 7$	______	______
(i) $f(x) = 0^x$	______	______
(j) $f(x) = \frac{1}{x}$	______	______
(k) $f(x) = 1^x$	______	______

III. Evaluate each of the following:

(a) $\{\!\{2.8\}\!\}$ (b) $[\![2.8]\!]$ (c) $\{\!\{-2.7\}\!\}$ (d) $\{\!\{\frac{1}{4}\}\!\}$

(e) $\lvert -20 \rvert$ (f) $\text{sgn } 2^{-3}$ (g) 4^3 (h) $\log_2 64$

(i) 5^2 (j) 5^{-2} (k) $[\![-100.4]\!]$ (l) $\log_{10} .01$

(m) $\{\{-10.3\}\}$ (n) $[[20]]$ (o) sgn -3 (p) sgn 0

(q) 0^0 (r) $|1^{-12}|$ (s) 0^3 (t) 1^{-11}

(u) $|\text{sgn } -4|$ (v) $\log_5 25$ (w) sgn 5 (x) $\log_{\frac{1}{2}} 4$

(y) 0^{-2} (z) 3^0

IV. For each of the following: A. Sketch the graph of the converse of the function; B. Give the domain of the converse; C. Give the range of the converse:

(a) $f(x) = \{\{x\}\}$ (b) $f(x) = \log_2 x$ (c) $f(x) = [[x]]$

(d) $f(x) = \frac{1}{x}$ (e) $f(x) = |x|$ (f) $f(x) = 2^x$

ANSWERS

I. (a) B., D. (b) A., B., C., E.,F,G (c) F (d) B., C., D., G., G.,H
(e) B., C., E., F. (f) B., C., D (g) G (h) C., G., H.

II. (a) $(-\infty, \infty)$; Integers (b) $(-\infty, \infty)$; $(0, \infty)$
(c) $(-\infty, \infty)$; $[0, \infty)$ (d) $(-\infty, \infty)$; $\{-1, 0, 1\}$
(e) $(0, \infty)$; $(-\infty, \infty)$ (f) $(-\infty, \infty)$; $[0, 1)$
(g) $(-\infty, \infty)$; $(-\infty, \infty)$ (h) $(-\infty, \infty)$; $\{7\}$
(i) $(0, \infty)$; $\{0\}$ (j) $\{x: x \neq 0\}$; $\{y: y \neq 0\}$
(k) $(-\infty, \infty)$; $\{1\}$

III. (a) .8 (b) 2 (c) .3 (d) .25 (e) 20 (f) 1
(g) 64 (h) 6 (i) 25 (j) $\frac{1}{25}$ (k) -101 (l) -2
(m) .7 (n) 20 (o) -1 (p) 0 (q) undefined (r) 1
(s) 0 (t) 1 (u) 1 (v) 2 (w) 1 (x) -2
(y) undefined (z) 1

IV. See your teacher if you have questions about the graphs.
(a) Domain: $[0,1)$ Range: $(-\infty, \infty)$
(b) Domain: $(-\infty, \infty)$; Range: $(0, \infty)$
(c) Domain: Integers; Range: $(-\infty, \infty)$
(d) Domain: $\{x: x \neq 0\}$; Range: $\{y: y \neq 0\}$
(e) Domain: $[0, \infty)$; Range: $(-\infty, \infty)$
(f) Domain: $(0, \infty)$; Range: $(-\infty, \infty)$