ALGEBRA 2

SYSTEMS OF LINEAR EQUATIONS

OBJECTIVES

I. Given a system of two linear equations in two variables

A. Solve the system
1. By graphing
2. By replacement
3. By linear combination

B. Identify the system as
1. Consistent
2. Inconsistent

C. Describe the relationship of the lines to each other as
1. Intersecting in one point
2. Coincident
3. Parallel

II. Solve a system of three equations in three variables

III. Solve word problems involving systems of equations

IV. Solve a system of equations in linear form where the variables appear as denominators.

SECTION ONE DEFINITIONS AND GRAPHICAL SOLUTIONS

Linear equation: an equation of the form $ax + by = c$ where a, b, and c a real numbers.

Linear system: a set of two linear equations.

Consistent system: a system of equations the graphs of which intersect i one point or in an infinite number of points.

Inconsistent system: a system of equations which graph as a pair of para lines.

Planar equation: an equation of the form $ax + by + cz = d$ where a, b, c, d are real numbers.

EXERCISE ONE

R EACH OF THE FOLLOWING: 1) GRAPH; 2) READ THE SOLUTION FROM THE GRA
3) IDENTIFY THE SYSTEM AS CONSISTENT OR INCONSISTENT
4) CHECK THE SOLUTION ALGEBRAICALLY

1. $x + 2y = 3$
$x = y$

2. $x + y = 3$
$-3x + 2y = 6$

3. $4x - y = 1$
$-4x + y = 8$

$3y = 2x + 2$
$6y = 4x + 12$

5. $2x + y = 9$
$x - y = 3$

6. $3x + y = 12$
$3x - 2y = -6$

$4x + 3y = 12$
$2x + 5y = 20$

8. $2x = y - 3$
$3y + 6 = x$

9. $0 = 2x - y + 3$
$4x - 2y = 2$

SECTION TWO

SOLUTION BY REPLACEMENT

Solution by replacement is also known as solution by substitution. The procedure is to solve one equation for one of the variables and then to replace that variable in the other equation by its equal.

For example:

Given:

Given:	$x - y = 2$	(1)
	$2x - 3y = -6$	(2)
	$x = 2 + y$	(3) solve (1) for x
	$2(2 + y) - 3y = -6$	(4) use (3) in (2)
	$4 - y = -6$	(5) simplify (4)
	$y = 10$	(6) solve (5) for y
	$x = 12$	(7) use (6) in (1)

Solution: (12,10)

EXERCISE TWO:

SOLVE EACH OF THE FOLLOWING SYSTEMS BY USING THE REPLACEMENT METHOD:

1. $x + y = 5$
$2x - y = 7$

2. $4x - y = 8$
$4x - 3y = 8$

3. $y = 2x$
$x + y = 6$

4. $8x + 5y = 21$
$x = y$

5. $x + y = 20$
$y = 2x + 5$

6. $y - 2x = 7$
$3x + 5y = 9$

NOTES: The replacement method is efficient if one of the variables in one of the equations has a coefficient of 1.

Always check the "solution" in both equations.

When listing the solution as an ordered pair, list the elements in alphabetical order.

SECTION THREE SOLUTION BY LINEAR COMBINATION

The linear combination method involves combining the equations under the operation of addition in order to have one of the variables fall out.

EXAMPLE:

$8x - 2y = 26$	(1)	
$4x + 2y = 22$	(2)	
$12x = 48$	(3)	(1) + (2)
$x = 4$	(4)	solve (3) for x
$16 + 2y = 22$	(5)	(4) in (2)
$2y = 6$		
$y = 3$		

SOLUTION: (4,3)

EXAMPLE:

$2x - 3y = 7$	(1)	
$6x + 2y = -1$	(2)	
$4x - 6y = 14$	(3)	(1) times 2
$18x + 6y = -3$	(4)	(2) times 3
$22x = 11$	(5)	(3) plus (4)
$x = .5$	(6)	solve (5) for x
$1 - 3y = 7$	(7)	(6) in (1)
$y = -2$	(8)	solve (7) for y

SOLUTION: (.5,-2)

EXERCISE THREE:

SOLVE EACH OF THE FOLLOWING SYSTEMS BY USING THE LINEAR COMBINATION METHOD:

1. $2x + y = 8$, $2x - y = 4$
2. $4x + y = -1$, $2x + 3y = 7$
3. $x = 3y + 3$, $2x = 9y + 5$
4. $3x - 2y = 12$, $4x + y = 5$
5. $5x = 4y + 2$, $x + 2y = -8$
6. $2x - 3y = -14$, $3x + 7y = 48$
7. $.5x + .2y = 1.3$, $.3x + .2y = 1.1$
8. $.3x + .4y = .6$, $x - 2y + 8 = 0$

NOTES: When copying the problem, arrange so that the same variables are under each other.

SECTION FOUR

THREE EQUATIONS WITH THREE UNKNOWNS

A system of three equations with three unknowns can generally be solved by carefully using the linear combination method. One usually must concentrate on the elimination of one of the variables twice in order to reduce the system to two equations with two unknowns. Study the following example:

$$3x + 2y - 10z = 7 \quad (1)$$
$$x + y + z = 9 \quad (2)$$
$$2x - y - 3z = -8 \quad (3)$$

Eliminate y

$$3x - 2z = 1 \quad (4)\ (2) + (3)$$
$$-2x - 2y - 2z = -18 \quad (5)\ (2) \text{ times } -2$$
$$x - 12z = -11 \quad (6)\ (1) + (5)$$

Use (4) & (6)

$$-3x + 36z = 33 \quad (7)\ (6) \text{ times } -3$$

$$34z = 34 \quad (8)\ (4) + (7)$$
$$z = 1 \quad (9) \text{ solve } (8) \text{ for } z$$

$$x = 1 \quad (10)\ (9) \text{ in } (4)$$

$$y = 7 \quad (11)\ (9) \text{ and } (10) \text{ in } (2)$$

The solution is the ordered triplet: (1,7,1)

TES: There are many ways to proceed through a problem. The above is just one of the ways.

It is very important to select one of the variables and eliminate it twice. Use equations in pairs; i.e. (1) & (2), (1) & (3), or (2) & (3). The result is a new system--two equations with two unknowns.

One tiny error will wreck a problem. Do be careful.

Check solutions in each of the three equations.

Express the solution as an ordered triplet...alphabetical order.

An equation in three variables represents a plane. To graph the equations one would need to use an X-axis, a Y-axis, and a Z-a

Systems of Linear Eq.

EXERCISE FOUR

SOLVE EACH OF THE FOLLOWING:

1. $$\begin{aligned} 2x + y + z &= 13 \\ x + 2y + z &= 11 \\ x + 2y + 3z &= 19 \end{aligned}$$

2. $$\begin{aligned} x + 3y + 4z &= 1 \\ y + z &= 1 \\ x - 2z &= 3 \end{aligned}$$

3. $$\begin{aligned} x + 2z &= 7 \\ 2x - y + 3z &= 9 \\ y - z &= 1 \end{aligned}$$

4. $$\begin{aligned} -x + y + 2z &= 3 \\ 2x - y + z &= 3 \\ -4x + 2y + 3z &= -1 \end{aligned}$$

5. $$\begin{aligned} 2x - 3y - 4z &= -21 \\ -4x + 2y - 3z &= -14 \\ -3x - 4y + 2z &= -10 \end{aligned}$$

6. $$\begin{aligned} 6x - y - 3z &= 2 \\ -3x + y - 3z &= 1 \\ -2x + 3y + z &= -6 \end{aligned}$$

7. $$\begin{aligned} 3x + 2y &= z - 7 \\ 5x + 3y &= -12 + 2z \\ 2x + 3y &= -5 + z \end{aligned}$$

8. $$\begin{aligned} 2x + 3y &= -2 \\ 4y + 2z &= -10 \\ 3x + 5z &= 1 \end{aligned}$$

9. $$\begin{aligned} 3x + 4z &= 20 \\ y - .5z &= 1 \\ 5x + 3y &= 19 \end{aligned}$$

10. $$\begin{aligned} 4x + y + z &= 5 \\ x - 2y - z &= 3 \\ 3x + 3y - 2z &= 22 \end{aligned}$$

SECTION FIVE — WORD PROBLEMS

For this section use the text assigned for the course. From ALGEBRA AND TRIGONOMETRY by Dolciani, Sorgenfrey, Brown, and Kane, see pages 291 and 292.

SECTION SIX — NON-LINEAR SYSTEMS -- VARIABLE IN THE DENOMINATOR

Use text as above. Page 288, #33-39.

NOTE: Use text pages 283 - 292 for general review.

TRIAL TEST

I. 1) Solve these systems of equations by graphing.
2) State the solution set.
3) Tell whether the system is consistent or inconsistent.
4) Check your roots.

(1) $x + y = 5$
$y = x + 1$

(2) $2x - y = 9$
$x + 2y = 7$

(3) $x + 2y = 6$
$x = 2 - 2y$

(4) $3x - 2y = 6$
$4y = 6x - 12$

(5) $x + 2y = 3$
$2x - 3y = 2$
$11x - y = -6$

II. Solve these algebraically:

(1) $2x + y = 9$
$x - y = 3$

(2) $z = 3t$
$3t - 6 = 2z$

(3) $2x - 4y = 12$
$y + \frac{1}{2}x = 3$

(4) $4x - 3y = 12$
$2x + 5y = 20$

(5) $3y - x = 6$
$\frac{1}{3}x - y = 2$

(6) $2x + 5y = 8$
$-3x + 7y = 17$

(7) $\frac{2}{x} + \frac{8}{y} = 10$

$\frac{5}{x} - \frac{4}{y} = 8$

(8) $\frac{2x}{3} + \frac{5y}{7} = \frac{-8}{7}$

$\frac{3x}{5} + \frac{7y}{10} = \frac{-4}{5}$

(9) $2x + y + z = 13$
$x + 2y + z = 11$
$x + 3y + 3z = 19$

(10) $a + 3b + 2c = 5$
$-3a + 2b + c = -6$
$4a + 4b + 3c = 13$

[illegible]

SYSTEMS OF LINEAR EQUATIONS

TRIAL RUN KEY

I. (1) (2,3) consistent

(2) (5,1) consistent

(3) $\emptyset$ inconsistent

(4) $\{(x,y): 3x - 2y = 6\}$ consistent

(5) (4,2) consistent

II. (1) (4,1)

(2) (-2,-6)

(3) $\{(x,y): 2x + 4y = 12\}$

(4) (0,4)

(5) no solution

(6) (-1,2)

(7) $\left(\frac{7}{6}, \frac{14}{13}\right)$

(8) (-6,4)

(9) (4,2,3)

(10) (3,4,-5)