MATH ANALYSIS-ANALYTIC GEOMETRY THE ELLIPSE

BEHAVIORAL OBJECTIVES

I. Define ellipse as

A. A locus of points from
   1. Two fixed points
   2. From a point and a line

B. A conic section

II. Identify the

A. Vertices of an ellipse
B. Foci of an ellipse
C. Major axis of an ellipse
D. Minor axis of an ellipse
E. Directrix of an ellipse
F. Eccentricity of an ellipse
G. Latus rectum of an ellipse

III. Given the equation of an ellipse determine

A. The center of the ellipse
B. The coordinates of the vertices of the ellipse
C. The coordinates of the foci of the ellipse
D. The equations of the directrices of the ellipse
E. The length of the
   1. Major axis
   2. Minor axis
F. The eccentricity of the ellipse
G. Its graph
H. The length of the latus rectum of the ellipse
I. The area of the ellipse
J. The perimeter of the ellipse

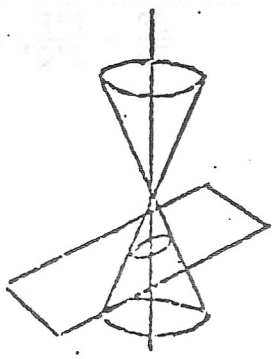
IV. Determine the equation of an ellipse given

A. The coordinates of the center and the length of the two axes.
B. The coordinates of the vertices and the foci.
C. The coordinates of the foci and the length of the major axis
D. The coordinates of the center and the foci
E. The coordinates of one focus, the equation of a directrix and the eccentricity
F. The coordinates of the endpoints of both axes

V. Solve problems involving properties of the ellipse.

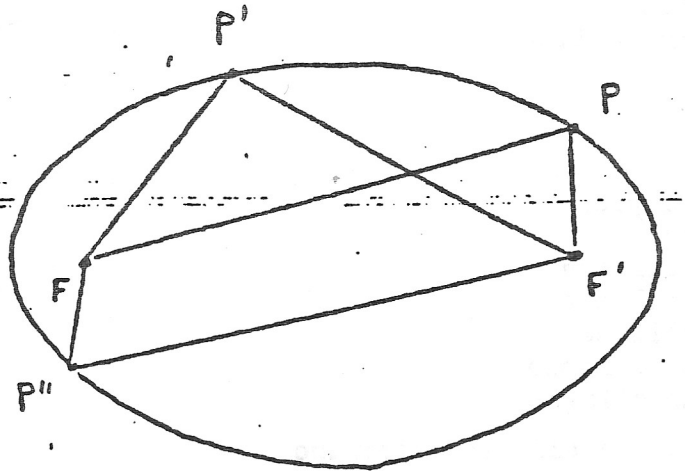
SECTION I ELLIPSE--DEFINITION

Ellipse--definition 1: An ellipse is the intersection of a plane and a right circular double-napped cone such that the plane cuts all elements of the cone.



The above definition will not be used algebraically. We include this definition to establish the ellipse as a conic section.

Ellipse--definition 2: An ellipse is the locus of points in a plane such that the sum of the distances from two fixed points to any point of the locus is a constant.

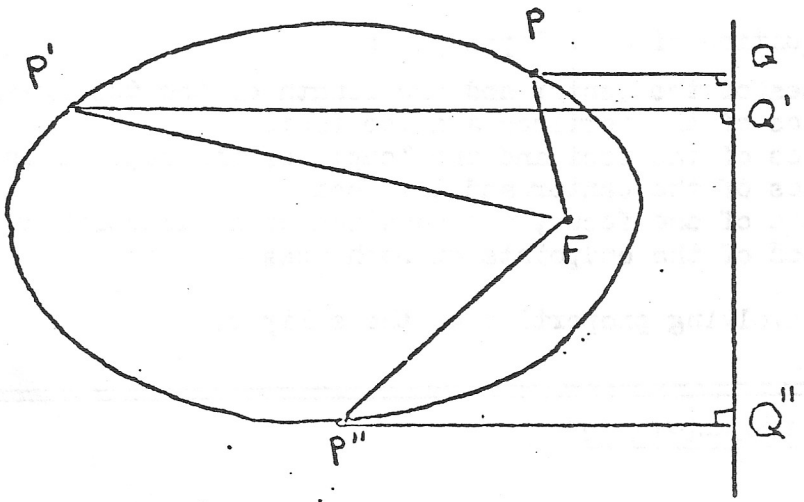


$FP + F'P = FP' + F'P' = FP'' + F'P'' =$ a constant. (Note: This constant must be greater than FF')

If F and F' are coincident we have the special ellipse called the circle!

Some joker might call an ellipse a "wall-eyed" circle!

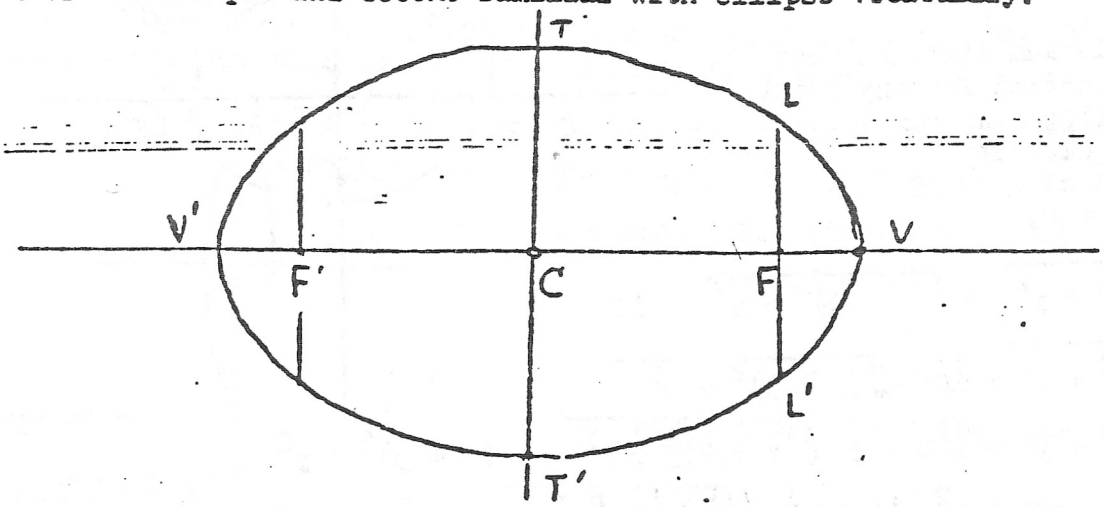
Ellipse--definition 3: An ellipse is the locus of points in a plane such that the ratio of the distance to a fixed point to the distance to a fixed line is some constant e. $0 < e < 1$. This ratio e is called the eccentricity of the ellipse.



$$\frac{PF}{PQ} = \frac{P'F}{P'Q'} = \frac{P''F}{P''Q''} = e$$

SECTION II THE ELLIPSE AS A LOCUS OF POINTS IN A PLANE EQUIDISTANT FROM TWO FIXED POINTS

Before we get to the algebra of the ellipse suppose we identify some special parts of the ellipse and become familiar with ellipse vocabulary.



The line VV' is called the major axis. It is customary to call the length of the segment VV' the length of the major axix. The length of the segment CV is the length of the semi-major axis. For the diagram above VV' is the major axis because of the two axes it is the longer.

TT' is called the minor axis. The length of the segment TT' is called the length of the minor axis. The length of the segment TC is called the length of the semi-minor axis.

F and F' are the foci of the ellipse. The foci are points on the major axis.

C is the center of the ellipse.

V and V' are the vertices of the ellipse.

The segment LL' is called the latus rectum of the ellipse. The line LL' is perpendicular to the major axis and contains a focus of the ellipse.

It can be shown with elementary calculus that the area of an ellipse is $\pi \cdot$ semi major axis $\cdot$ semi minor axis. (Note if both axes are equal in length the figure is a circle and the area is our familiar πr^2.

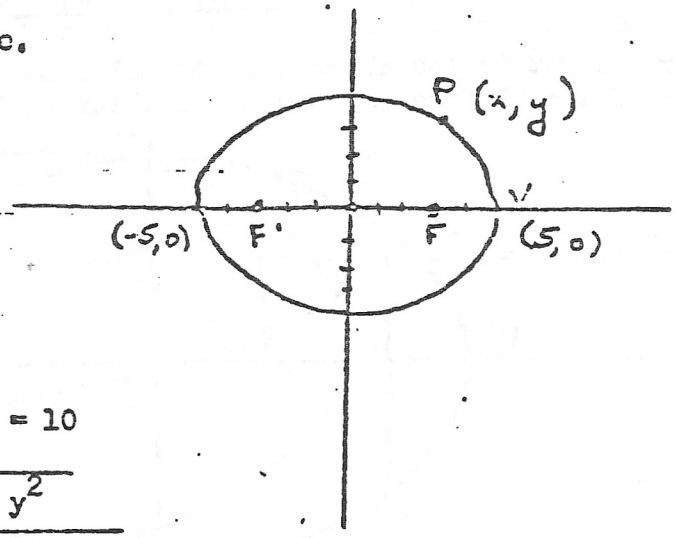
It can be shown by not so elementary calculus that the perimeter of an ellipse is $2\pi\sqrt{\tfrac{1}{2}(\text{semi-major axis}^2 + \text{semi-minor axis}^2)}$. (Note again if the ellipse is a circle and both axes are congruent, the formula simplifies the the familiar $2\pi r$.)

NOTICE: $TF = TF' = CV = CV'$

Now an example and some algebra!

Given: An ellipse with center (0,0), foci at (3,0) and (-3,0), and vertices at (5,0) and (-5,).

Determine the equation of the ellipse.



$VF + VF' = 10$ (Why?)

Since this sum distance must remain constant for any point on the ellipse we can set up the equation:

$PF + PF' = 10$

$$\sqrt{(x-3)^2 + y^2} + \sqrt{(x+3)^2 + y^2} = 10$$

$$\sqrt{(x-3)^2 + y^2} = 10 - \sqrt{(x+3)^2 + y^2}$$

$$(x-3)^2 + y^2 = 100 - 20\sqrt{(x+3)^2 + y^2} + (x+3)^2 + y^2$$

$$x^2 - 6x + 9 + y^2 = 100 - 20\sqrt{(x+3)^2 + y^2} + x^2 + 6x + 9 + y^2$$

$$20\sqrt{(x+3)^2 + y^2} = 100 + 12x$$

$$5\sqrt{(x+3)^2 + y^2} = 25 + 3x$$

$$25x^2 + 150x + 225 + 25y^2 = 625 + 150x + 9x^2$$

$$16x^2 + 25y^2 = 400$$

$$\boxed{\frac{x^2}{25} + \frac{y^2}{16} = 1}$$

This is the informational form for the equation of the ellipse above.

The length of the major axis (the longer one) is 10. It is two times the square root of the denominator of the x^2 term.

The length of the minor axis (the shorter one) is 8. It is two times the square root of the denominator of the y^2 term.

The center is at (0,0). The foci are at (3,0) and (-3,0).

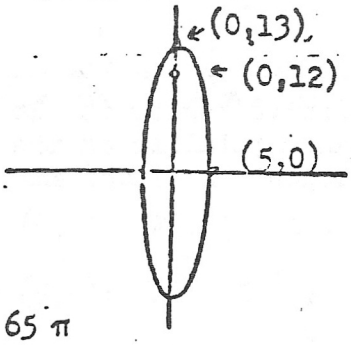
The vertices are at (5,0) and (-5,0).

In the above diagram WHEN $PF = PF'$, the $PF = 5$. The distance from the endpoint of the minor axis to a focus is equal to the length of the semi-major axis. We also have a nice right triangle brewing. It is customary to call the distance from the center to the focus "c". This number c is the focal distance.

NOTE: $c^2 + (\text{semi-minor axis})^2 = (\text{semi-major axis})^2$

Suppose $\frac{x^2}{25} + \frac{y^2}{169} = 1$

The above is an ellipse. It is centered at (0,0). The easy way to get the X and Y intercepts is to first let x = 0 and solve for y. Then let y = 0 and solve for x. Thus we determine that the graph contains the points (0,13), (0,-13), (5,0), and (0,-5). Actually, this is enough to sketch the graph.



Area: 65π

Perimeter: $2\pi\sqrt{72}$

The length of the major axis is 26.

The length of the minor axis is 10.

The foci, this time, are on the vertical axis.

$$c^2 + 25 = 169$$
$$c^2 = 144$$
$$c = \pm 12$$

The coordinates of the foci are (0,12) and (0,-12).

To find the length of the latus rectum let y = 12 and solve for x.

$$\frac{x^2}{25} + \frac{144}{169} = 1$$
$$x^2 = 25\left(\frac{25}{169}\right)$$
$$|x| = \frac{25}{13}$$

Hence, the length of the latus rectum is $\frac{50}{13}$.

EXERCISE 1

For numbers 1 to 8, (a) Graph; (b) Determine the length of the major axis; (c) Determine the length of the minor axis; (d) Find the coordinates of the vertices; (e) Find the coordinates of the foci; (f) Determine the length of the latus rectum; (g) Find the area; (h) Find the perimeter.

1. $\frac{x^2}{4} + \frac{y^2}{9} = 1$

2. $\frac{x^2}{9} + \frac{y^2}{100} = 1$

3. $\frac{x^2}{64} + y^2 = 1$

4. $\frac{x^2}{5} + \frac{y^2}{25} = 1$

5. $x^2 + 4y^2 = 100$

6. $x^2 = 64 - 4y^2$

7. $\frac{9x^2}{16} + 25y^2 = 1$

8. $64x^2 + 4y^2 = 1$

9. Write the equation of the ellipse given:

(a) Vertices (5,0) and (-5,0) and minor axis of length 4.

(b) Vertices (0,8) and (0,-8) and minor axis of length 3.

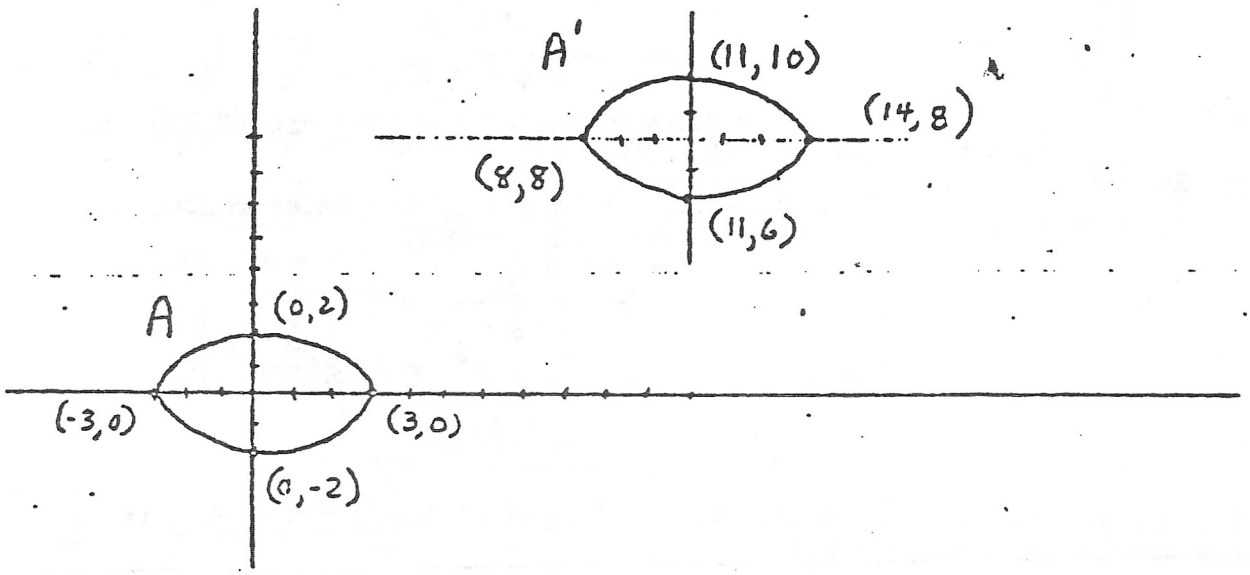
(c) Center at (0,0), minor axis of length 2 and major axis of length 5.

(d) Vertices at (0,13) and(0,-13) and foci at (0,5) and (0,-5).

(e) Vertices at (13,0) and (-13,0) and foci at (12,0) and (-12,0).

SECTION III. THE ELLIPSE CENTERED AT (h,k)

The equation for the ellipse centered at (h,k) can be derived using the locus formula. The algebra is good practice. However, we can capitalize on the equation for the ellipse centered at (0,0). Study the diagram below:



Ellipse A and Ellipse A' are congruent. The equation for ellipse A is

$$\frac{x^2}{9} + \frac{y^2}{4} = 1.$$

Notice, the x coordinate of each point on Ellipse A' is 11 units larger than its corresponding point on ellipse A. Similarly, the y coordinate of each point on ellipse A' is 8 units larger than its corresponding point on ellipse A.

The equation of ellipse A' is $\frac{(x-11)^2}{9} + \frac{(y-8)^2}{4} = 1$

In general, for an ellipse centered at (h,k) the equation is:

$$\boxed{\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1}$$

(h,k) is the center. $|a|$ is the length of the semi-horizontal axis. $|b|$ is the length of the semi-vertical axis.

Notice again, if a = b, we have the equation of a circle.

Exercise 2

I. For each of the following: (a) Determine the coordinates of the center; (b) Find the length of the major axis; (c) Find the length of the minor axis; (d) Find the coordinates of the vertices; (e) Find the coordinates of the foci. (f) Graph.

1. $\frac{(x+2)^2}{4} + \frac{(y-3)^2}{9} = 1$

2. $\frac{(x-7)^2}{25} + \frac{(y-1)^2}{16} = 1$

3. $\frac{9(x+2)^2}{25} + 4(y-3)^2 = 1$

4. $5x^2 + 2y^2 - 10x + 4y = 3$

II. Write the equation of each of the following ellipses:

1. F: (4,0); F': (-4,0), major axis of length 14.

2. F: (9,2); F': (5,2); major axis of length 6.

3. Major axis along the X-axis and of length 10. Minor axis of length 6. Center at (3,0).

4. F:(3,3); F': (3,-9); V: (3,7); and V': (3,-13)

III. Write each of the following ellipse equations in the informational form:

1. $4x^2 + y^2 + 6y - 9 = 0$

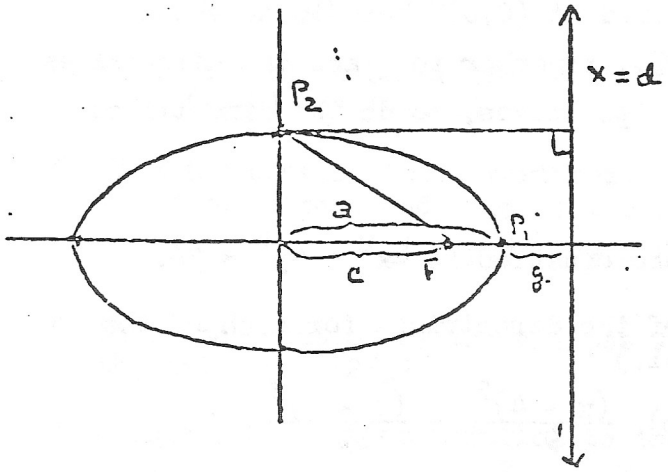
2. $2x^2 + 8x + 3y^2 + 9y + 2 = 0$

3. $2x^2 + 3x + y^2 - 7y - 4 = 0$

SECTION III — ECCENTRICITY AND THE DIRECTRIX

In this section we shall devote some time to studying the ellipse as it is defined in terms of eccentricity. Recall definition 3: An ellipse is the locus of points in a plane such that the ratio of the distance to a fixed point to the distance to a fixed line is some constant e. $0 < e < 1$. e is called the eccentricity of the ellipse.

Study the diagram below:



$P_1F = a - c$ $\quad P_1$ to $d = g$

$P_2F = a$ $\quad P_2$ to $d = g + a$

Hence: $\frac{a-c}{g} = \frac{a}{g+a}$

So: $ag - cg + a^2 - ac = ag$

$a^2 - ac = cg$

$a(a-c) = cg$

$\frac{a-c}{g} = \frac{c}{a}$

The eccentricity of the ellipse is $\frac{c}{a}$ ...for an ellipse with major axis horizontal.

The eccentricity will similarly be $\frac{c}{b}$ for an ellipse with major axis vertical.

The ellipse has two directrices. They are parallel to the minor axis. The directrix is the fixed line referred to in the just previous definition.

For the diagram on the previous page, the equations of the directrices are:

$$x = a + g$$

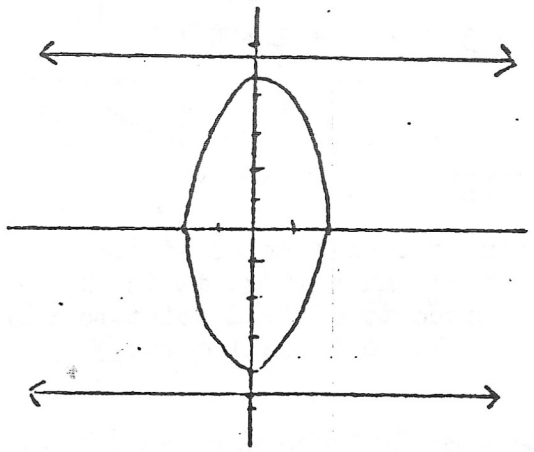
$$x = a + \frac{a^2 - ac}{c}$$

$$x = \frac{ac + a^2 - ac}{c}$$

$$x = \frac{a^2}{c}$$

Since the ellipse is symmetric around its minor axis, there are two directrices, one at $x = a^2/c$ and the other at $-a^2/c$.

EXAMPLE: Consider the ellipse with equation : $\frac{x^2}{4} + \frac{y^2}{16} = 1$



The length of the semi-major axis is 4.
The length of the semi-minor axis is 2.

The eccentricity of the ellipse is

$$\frac{\sqrt{12}}{4} \text{ or } \frac{\sqrt{3}}{2}$$

The equation of the directrices are

$$y = \pm \frac{16}{\sqrt{12}} \text{ or } y = \pm \frac{8\sqrt{3}}{3}$$

IMPORTANT NOTICE: If the graph is not centered at (0,0) there is no change in the eccentricity...BUT...remember to place the directrices accordingly. If the ellipse moves, so do the directrices!

EXERCISE 3

1. Find the eccentricity of the ellipse whose equation is $4x^2 + 9y^2 = 36$.

2. Find the eccentricity and the equation of the directrices for each of the following: (A sketch might prove helpful.)

(a) $25x^2 + 16y^2 = 400$

(b) $\frac{(x-4)^2}{9} + \frac{(y-5)^2}{25} = 1$

(c) $x^2 + 3y^2 = 4$

(d) $x^2 + 2y^2 = 1$

(e) $(x+1)^2 + \frac{(y-3)^2}{4} = 1$ (f) $\frac{(x-7)^2}{64} + \frac{(y-3)^2}{16} = 1$

3. Determine the equations of the following ellipses:

(a) Focus at (3,0), directrix: $x = \frac{25}{3}$, and eccentricity = $\frac{3}{5}$.

(b) Focus at (1,2), directrix $y = -2$, and eccentricity = $\frac{1}{2}$.

ANALYTIC GEOMETRY STUDENTS: Review all work carefully. Become an expert on the basics of the ellipse. Take the trial run and then take the test.

MATH ANALYSIS STUDENTS: Go on to the final section.

SECTION 4 FUN STUFF

There are really no new concepts developed in this section. It merely calls upon you to explore the ellipse in more complicated situations. Enjoy the exercises!

EXERCISE 4

1. Write the equation of the ellipse whose center is at the origin, whose axes of symmetry are the coordinate axes, and whose major axis is three times the minor axis. The point (3,1) is on the ellipse. (There are two solutions.)

2. The endpoints of one side of a triangle are (-2,0) and (2,0) and the sum of the lengths of the other two sides is 6. Write the equation of the locus of points which are possible vertices of the triangle.

3. Write the equation of the circle which passes through the endpoints of the latera recta (that's plural for latus rectum) of the ellipse whose equation is: $\frac{x^2}{16} + \frac{y^2}{12} = 1$.

4. Write the equation of the locus of points such that the sum of the distances from (5,-1) and (-1,-1) is 10.

5. Find the center, foci, vertices, endpoints of the later recta, equations of the directrices, and area of the ellipse with equation:

$$x^2 + 2y^2 + 2x - 12y + 11 = 0$$

6. Find the equation of the ellipse with vertices (-5,0), (5,0) which contains the point (4, 12/5).

7. A square with sides parallel to the coordinate axes is inscribed in the ellipse with equation $9x^2 + 16y^2 = 100$. Find the coordinates of the vertices of the square. What is the area of the square? What is the equation of the circles circumscribed around the square?

8. Sketch an ellipse which is tangent to the coordinate axes and has a center at (-5,9). Write the equation of the ellipse.

9. Write the equation of the ellipse with major axis of length 10 and foci at (8,2) and (0,2).

10. Write the equation of the ellipse whose center is at the origin, whose axes are on the coordinate axes and contains the points (2,2) and (3,-1).

11. Write the equation of the ellipse with center (4,5), minor axis 12 units long and area 48π sq. units.

12. An arch is in the form of a semi-ellipse, with the major axis as the span. If the span is 80 feet long and the height is 30 feet, find the height of the arch at a point 20 feet from the minor axis.

13. Find the equation of the locus of points which are twice as far from the line $y = 10$ as from the point (0,4).

14. Prove that the measure of the minor axis of an ellipse is the mean proportional between the length of the major axis and the length of the latus rectum.

THAT'S ALL. Go on to the Trial Run and the Test.

ANSWERS

EXERCISE 1:

1. (b) 6; (c) 4; (d) $(0,\pm 3)$; (e) $(0,\pm\sqrt{5})$; (f) $\frac{4}{3}$; (g) 6π; (h) $\pi\sqrt{26}$
2. (b) 20; (c) 6; (d) $(0,\pm 10)$; (e) $(0,\pm\sqrt{91})$; (f) 9/5; (g) 30π; (h) $2\pi\sqrt{54.5}$
3. (b) 16; (c) 2; (d) $(\pm 8,0)$; (e) $(\pm\sqrt{63},0)$; (f) $\frac{1}{4}$; (g) 8π; (h) $\pi\sqrt{130}$
4. (b) 10; (c) $2\sqrt{5}$; (d) $(0,\pm 5)$; (e) $(0,\pm\sqrt{20})$; (f) 2; (g) $5\sqrt{5}\pi$ (h) $2\pi\sqrt{15}$
5. (b) 20; (c) 10; (d) $(\pm 10,0)$; (e) $(\pm 5\sqrt{3},0)$; (f) 5; (g) 50π; (h) $\pi\sqrt{250}$
6. (b) 16; (c) 8; (d) $(\pm 8,0)$; (e) $(\pm 4\sqrt{3},0)$; (f) 4; (g) 32π; (h) $4\pi\sqrt{10}$
7. (b) $\frac{8}{3}$; (c) $\frac{2}{5}$; (d) $(\pm\frac{4}{3},0)$; (e) $(\pm\frac{\sqrt{391}}{15},0)$; (f) $\frac{3}{50}$; (g) $\frac{4\pi}{15}$; (h) $\frac{\pi}{15}\sqrt{31.8}$
8. (b) 1; (c) $\frac{1}{2}$; (d) $(0,\pm\frac{1}{2})$; (e) $(0,\pm\frac{\sqrt{15}}{8})$; (f) $\frac{1}{16}$; (g) $\frac{\pi}{16}$; (h) $\frac{\pi}{8}\sqrt{34}$
9. (a) $\frac{x^2}{25}+\frac{y^2}{4}=1$; (b) $\frac{4x^2}{9}+\frac{y^2}{64}=1$; (c) $x^2+\frac{4y^2}{25}=1$

   (d) $\frac{x^2}{144}+\frac{y^2}{169}=1$; (e) $\frac{x^2}{169}+\frac{y^2}{25}=1$ OR (c) $\frac{4x^2}{25}+y^2=1$

NOW THAT YOU HAVE DONE THE WORK DO YOU NOTICE THAT THE LENGTH OF THE LATUS RECTUM IS: $2(\text{semi-minor axis})^2/(\text{semi-major axis})$

ELLIPSE ANSWERS CONTINUED:

EXERCISE 2

1. (a) $(-2,3)$; (b) 6; (c) 4; (d) $(-2,0)$ and $(-2,6)$; (e) $(-2, 3 \pm \sqrt{5})$.

2. (a) $(7,1)$; (b) 10; (c) 8; (d) $(2,1)$ and $(12,1)$; (e) $(10,1)$ and $(4,1)$.

3. (a) $(-2,3)$; (b) $\frac{10}{3}$; (c) 1 (d) $(-\frac{1}{3},3)$ and $(-\frac{11}{3},3)$; (e) $(2 \pm \frac{\sqrt{91}}{6},3)$.

4. (a) $(1,-1)$; (b) $2\sqrt{5}$; (c) $2\sqrt{2}$; (d) $(1,-1 \pm \sqrt{5})$; (e) $(1,-1 \pm \sqrt{3})$

II. 1. $\frac{x^2}{49} + \frac{y^2}{33} = 1$

2. $\frac{(x-7)^2}{9} + \frac{(y-2)^2}{5} = 1$

3. $\frac{(x-3)^2}{25} + \frac{y^2}{9} = 1$

4. $\frac{(x-3)^2}{64} + \frac{(y+3)^2}{100} = 1$

III. 1. $\frac{2x^2}{9} + \frac{(y+3)^2}{18} = 1$

2. $\frac{8(x+2)^2}{51} + \frac{4(y+\frac{3}{2})^2}{17} = 1$

3. $\frac{16(x+\frac{3}{4})^2}{139} + \frac{8(y-\frac{7}{2})^2}{139} = 1$

EXERCISE 3

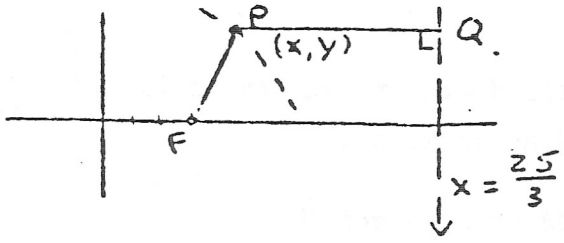
1. $\frac{\sqrt{5}}{3}$.

2. (a) $\frac{3}{5}$; $y = \pm \frac{25}{3}$; (b) $\frac{4}{5}$; $y = 5 \pm \frac{25}{4}$

(c) $\frac{\sqrt{6}}{3}$, $x = \pm\sqrt{6}$; (d) $\frac{\sqrt{2}}{2}$, $x = \pm\sqrt{2}$; (e) $\frac{\sqrt{3}}{2}$, $y = 3 \pm \frac{4\sqrt{3}}{3}$

(f) $\frac{\sqrt{3}}{2}$, $x = 7 \pm \frac{16\sqrt{3}}{3}$

3. To solve this set use the locus definition of the ellipse.



$\frac{PF}{PQ} = \frac{3}{5} \Rightarrow 5\,PF = 3\,PQ$

$5\sqrt{(x-3)^2 + y^2} = 3(\frac{25}{3} - x)$

The above is a good answer. However, it can be simplified to:

$\frac{x^2}{25} + \frac{y^2}{16} = 1$

(b) Procede as above. When simplified the answer is:

$\frac{3(x-1)^2}{16} + \frac{9(y-\frac{10}{3})^2}{64} = 1$

ELLIPSE ANSWERS CONTINUED

EXERCISE 4

1. $9x^2 + y^2 = 82$ or $x^2 + 9y^2 = 18$

2. $\frac{x^2}{9} + \frac{y^2}{5} = 1$ $(y \neq 0)$

3. $x^2 + y^2 = 13$

4. Ellipse locus. Foce (5,-1) and (-1,-1). Center (2,-1). The major axis is 10. The major axis is horizontal. The minor axis is vertical and is equal in length to 4. Equation: $\frac{(x-2)^2}{25} + \frac{(y+1)^2}{16} = 1$

5. Center (-1,3). Foci: (1,3) and (-3,3). Vertices: $(-1, \pm 2\sqrt{2}, 3)$. Endpoints of latera recta: $(1, 3 \pm \sqrt{2})$ and $(-3, 3 \pm \sqrt{2})$ Directrices: $x = -5$, $x = 3$; Area: $4\pi\sqrt{2}$.

6. $\frac{x^2}{25} + \frac{9y^2}{144} = 1$

$\frac{x^2}{25} + \frac{y^2}{16} = 1$

7. Vertices of square: $(2, \pm 2)$; $(-2, \pm 2)$
Area of square: 16 sq. units
Equation of the circle: $x^2 + y^2 = 8$

8. $\frac{(x+5)^2}{25} + \frac{(y-9)^2}{81} = 1$

9. $\frac{(x-4)^2}{25} + \frac{(y-2)^2}{9} = 1$

10. $\frac{3x^2}{32} + \frac{5y^2}{32} = 1$

11. $\frac{(x-4)^2}{64} + \frac{(y-5)^2}{36} = 1$ or $\frac{(x-4)^2}{36} + \frac{(y-5)^2}{64} = 1$

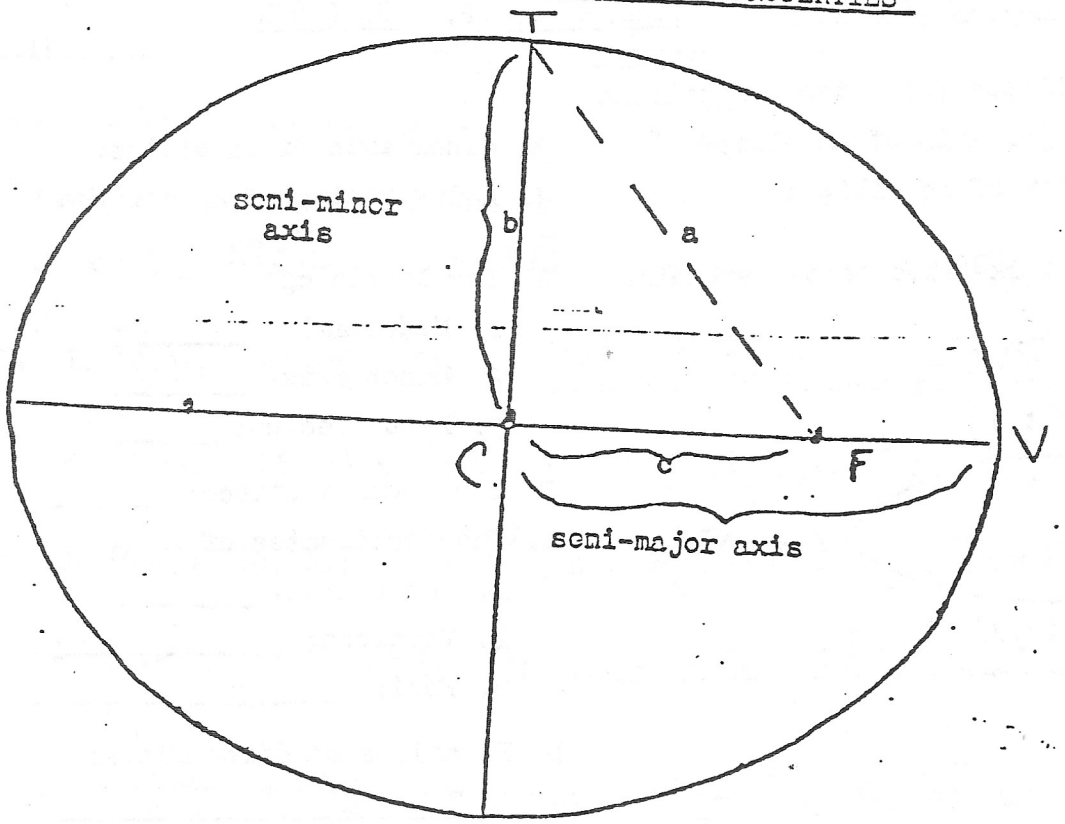
12. $15\sqrt{3}$

13. $\frac{x^2}{12} + \frac{(y-2)^2}{16} = 1$

14. For the ellipse with equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\frac{x}{2b} = \frac{2b}{2a}$

SUMMARY: BASIC ELLIPSE PROPERTIES



Note: CV = a

Area = $\pi a b$

Perimeter = $2\pi \sqrt{\frac{1}{2}(a^2 + b^2)}$

CF = focal distance = c

c^2 + (semi-minor axis)^2 = (semi-major axis)^2

Length of latus rectum = $\frac{2(\text{semi-minor axis})^2}{\text{semi-major axis}}$

[Latus rectum is a chord of the ellipse which contains a focus.]

Eccentricity = $\frac{c}{\text{semi-major axis}}$

Directrices: Major axis horizontal: $x = h \pm \frac{(\text{semi-major axis})^2}{c}$

Center at (h,k)

Major axis vertical: $y = k \pm \frac{(\text{semi-major axis})^2}{c}$

Center at (h,k)

MATH ANALYSIS-ANALYTIC GEOMETRY ELLIPSE L.A.P. TRIAL RUN

I. Define: 1. Ellipse (Give two definitions)

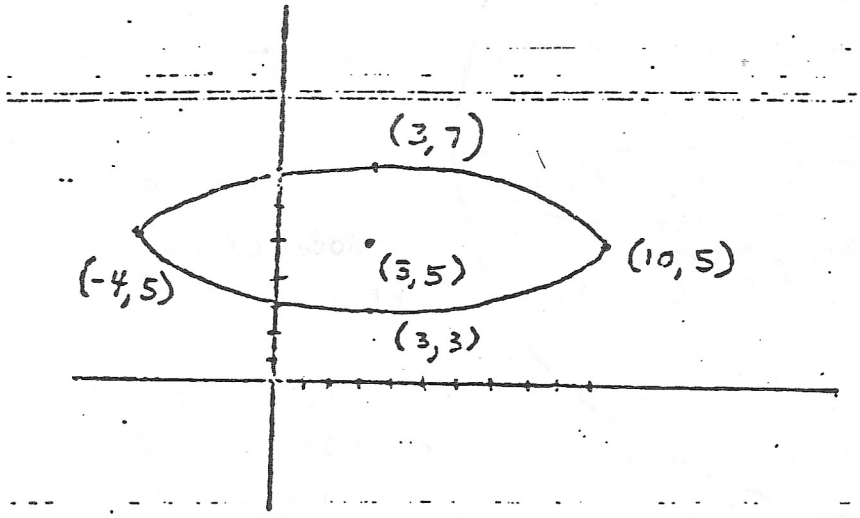
2. Major axis of an ellipse

3. Minor axis of an ellipse

4. Foci of an ellipse

5. Latus rectum of an ellipse

II. For the ellipse sketched below determine:



A. The length of

1. Major axis: _____

2. Minor axis: _____

3. Latus rectum: _____

B. The focal distance: _____

C. The coordinates of

1. The center: _____

2. Vertices: _____

3. Foci: _____

D. Equations of directrices:

_____

E. Eccentricity: _____ G. Area: _____ H. Perimeter: _____

III. For each of the following: Graph and determine:

A. Center; B: Vertices; C. Foci: D. Eccentricity; E. Directricies.

1. $\frac{x^2}{9} + \frac{y^2}{49} = 1$

2. $\frac{(x-1)^2}{25} + (y+3)^2 = 1$

3. $25x^2 + y^2 = 36$

4. $9x^2 + 4y^2 - 36x + 24y + 36 = 0$

IV. Determine the equationof each ellipse determined below:

1. Center (5,8); Major axis 10; Minor axis 4. (Two solutions)

2. Vertices (8,0) and (-8,0); eccentricity = 3/4.

3. Foci at (3,7) and (3,11); Major axis of length 12.

4. Axes on the coordinate axes. (5,2) and (2,8) on the ellipse.

V. MATH ANALYSIS STUDENTS: Review all problems from the last section of the L.A.P.

ELLIPSE L.A.P. TRIAL RUN ANSWERS

II. A. 1. 14; 2. 4; 3. $\frac{8}{7}$. B. $\sqrt{45}$

C. 1. (3,5); 2. (-4,5) and (10,5); 3. $(3 \pm \sqrt{45}, 5)$

D. $x = 3 \pm \frac{49\sqrt{5}}{15}$ E. $\frac{3\sqrt{5}}{7}$; G. 14π; H. $\pi\sqrt{106}$

III. 1. (0,0), $(0,\pm 7)$; $(0, \pm 2\sqrt{10})$; $\frac{2\sqrt{10}}{7}$; $x = \pm \frac{49\sqrt{10}}{20}$

2. (1,-3); (6,-3) and (-4,-3); $(1 \pm 2\sqrt{6}, -3)$; $\frac{2\sqrt{6}}{5}$; $x = 1 \pm \frac{25\sqrt{6}}{12}$

3. (0,0); $(0,\pm 6)$; $(\pm \frac{12\sqrt{6}}{5}, 0)$; $\frac{2\sqrt{6}}{5}$; $y = \pm \frac{5\sqrt{6}}{2}$

4. (2,-3); (2,0) and (2,-6); $(2, -3 \pm \sqrt{5})$; $\frac{\sqrt{5}}{3}$; $y = -3 \pm \frac{9\sqrt{5}}{5}$

IV. 1. $\frac{(x-5)^2}{25} + \frac{(y-8)^2}{4} = 1$ and $\frac{(x-5)^2}{4} + \frac{(y-8)^2}{25} = 1$

2. $\frac{x^2}{64} + \frac{y^2}{28} = 1$ 3. $\frac{(x-3)^2}{32} + \frac{(y-9)^2}{36} = 1$

4. $\frac{5x^2}{132} + \frac{7y^2}{528} = 1$