MATH ANALYSIS--ANALYTIC GEOMETRY

EQUATIONS AND FUNCTIONS

BEHAVIORAL OBJECTIVES

I. Solve equations of the following types

- A. Linear
- B. Quadratic
- C. Absolute value
- D. Exponential
- E. Greatest Integer
- F. Logarithmic

II. Use basic algebra to solve for a given variable in a standard formula

III. Solve inequalities of the following types

- A. Linear
- B. Quadratic
- C. Polynomial
- D. Absolute value
- *E. Prove some basic inequality theorems

IV. Given a polynomial function $f(x)$, determine

- A. The values of x for which $f(x) = 0$
- B. The values of x for which $f(x) > 0$
- C. The values of x for which $f(x) < 0$
- D. The degree of the polynomial
- E. Whether the function is
  1. Even
  2. Odd
  3. Neither
- F. The graph of the function

*V. Describe the graph of a polynomial function $f(x)$ when the function is given in general terms such as $f(x) = k(x - a_1)(x - a_2)(x - a_3)$.

SECTION I EQUATIONS

This section calls to mind the various methods used to solve equations of different types. By and large, most techniques used are developed in courses such as Algebra 1 and Algebra 2. Some examples will be given and then the reader is ~~asked to give the exercises~~ careful attention.

Example 1: A linear equation. The highest power of the variable is 1.

Solve for x: $3x - 15 = 6x + 10$

$-3x - 15 = 10$ [Subtract 6x from both sides]

$-3x = 25$ [Add 15 to both sides]

$x = -\frac{25}{3}$ [Divide both sides by -3]

---

* Math Analysis only

Example 2: The quadratic equation. The highest degree of the variable is 2.

A quadratic equation can be solved in one of three ways:

1. Factoring; 2. Completint the square 3. Using the quadratic formula. For $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$b^2 - 4ac$ is called the discriminant.

A quadratic equation can have no real solutions, one real solution, or two real solutions.

If the value of the discriminant is < 0, the equation has no real solutions.

Solve for x: $x^2 + 4x - 5 = 0$

A. By factoring:

$(x + 5)(x - 1) = 0$

$x = -5$ or $x = 1$

B. By completing the square:

$x^2 + 4x + 4 = 5 + 4$

$(x + 2)^2 = 9$

$|x + 2| = 3$

$x + 2 = 3$ or $x + 2 = -3$

$x = 1$ or $x = -5$

C. By using the quadratic formula:

$$x = \frac{-4 \pm \sqrt{16 + 20}}{2}$$

$$x = \frac{-4 \pm \sqrt{36}}{2} = \frac{-4 \pm 6}{2}$$

$x = 1$ or $x = -5$

Example 3: An equation having an absolute value:

A. $|2x + 4| = 4$

$2x + 4 = 4$ or $2x + 4 = -4$

$x = 0$ or $x = -4$

B. $|3x - 5| = |7x - 2|$

$3x - 5 = 7x - 2$

$-4x = 3$

$x = -3/4$

or: $3x - 5 = -7x + 2$

$10x = 7$

$x = 7/10$

Example 4: Exponential and Logs.

Recall: $\log_b a = x \iff b^x = a$ [Definition of log.]

$\log a + \log b = \log ab$ [Product rule]

$\log a - \log b = \log a/b$ [Quotient rule]

$\log a^b = b \log a$ [Power rule]

Solve for x:

A. $2^{3x-1} = 64$

$2^{3x-1} = 2^6$

$3x - 1 = 6$

$3x = 7$

$x = 7/3$

B. $5^{2x} = 7$

$\log 5^{2x} = \log 7$ [Take the log of both sides]

$2x \log 5 = \log 7$ [Power rule]

$x = \frac{\log 7}{2 \log 5} \doteq .6$ [Algebra]

C. $\log (x + 1) + \log (x - 2) = 1$

$\log (x + 1)(x - 2) = 1$ [Product rule]

$x^2 - x - 2 = 10$ [Definition of log]

$x^2 - x - 12 = 0$

$(x - 4)(x + 3) = 0$

$x = 4;\ x = -3$ [Reject $x = -3$ because the domain of the log function is the set of positive real numbers.]

Example 5: Greatest Integer.

$[\![2x - 7]\!] = 8$ The greatest integer that does not exceed $2x - 7$ is 8.

$8 \leq 2x - 7 < 9$ Hence, $2x - 7$ must be greater than or equal to 8 and less than 9.

$15 \leq 2x < 16$

$7.5 \leq x < 8$

The exercises cited above give a brief overview of some techniques needed to do the work in Exercise 1. After going over the examples, work carefully through Exercise 1. The management wishes you good luck and some hours of sheer delight!

EXERCISE 1

1. $3x - 2(x + 1) = \frac{1}{2}(2x + 10)$

2. $\frac{x + 2}{25} = \frac{x - 1}{15}$

3. $[\![5x + 1]\!] = -2$

4. $|x^2 + 5x + 4| = 0$

5. $|x - 5| = |x + 3|$

6. $3^{7x} = 27$

7. $x^2 + 7x = 12$

8. $x^4 + 3x^2 + 2 = 0$

9. $\frac{3}{x + 2} + 4 = \frac{2}{4x + 8}$

10. $|2x + 1| = 3x - 1$

11. $\sqrt{x - 5} = 8$

12. $|-2x - 15| = 6$

13. $\frac{x + 2}{4} - \frac{x - 3}{3} = \frac{1}{2}$

14. $\frac{4}{3x} = \frac{3}{2x - 1}$

15. $\sqrt{x^2 + 27} - 3 = x$

16. $3^x = \frac{1}{27}$

17. $\log_x 8 = 3$

18. $x^{3/2} = 125$

19. $3x^2 + 2x - 4 = 0$

20. $\frac{3}{x + 4} - \frac{7}{x - 4} = \frac{8}{x^2 - 16}$

21. $\sqrt{x - 5} = 4$

22. $3x(x - 6) = 0$

23. $x^2 - 16 = 0$

24. $\log (x + 2) - \log (x + 1) = 1$

25. $\log (x + 2) + \log (x + 1) = \log 2$

26. $\log_4(x - 3) = 2$

27. $\log 2x + \log x = 2$

28. $4^x = 32$

29. $[\![2x - 1]\!] = x$

30. $2^{x+1} = 128$

31. $4x^3 = 2x - 7x^2$

32. $\frac{3}{x^2} - \frac{5}{x} = -2$

33. $\log x^2 = \log x$

34. $\log_4(x - 3) = 2$

35. $\log x^2 - \log 5 = \log 7 + \log 2x$

36. $\log_{27} x = \frac{1}{3}$

37. $\log_2(x^2 + 3x + 4) = 1$

38. $5^x = 4$

39. $4^{3-x} = 2$

40. $\sqrt{2x - 3} - \sqrt{x - 2} = 1$

SECTION 2 TRANSFORMING FORMULAS

In both mathematics and science a variety of formulas are used. At times is is necessary to transform a formula to solve for a particular variable.

For example: The formula for the volume of a sphere is:

$$V = \frac{4}{3}\pi r^3$$

Suppose the volume is known and the task is to determine the value of the radius. Solve for r:

$$V = \frac{4}{3}\pi r^3$$

$$3V = 4\pi r^3$$

$$\frac{3V}{4\pi} = r^3$$

$$\sqrt[3]{\frac{3V}{4\pi}} = r$$

EXERCISE 2 For each of the following formulas, solve for the indicated variable.

1. $P = 2L + 2W$; W

2. $S_n = \frac{n(a_1 + a_n)}{2}$; a_n

3. $V = \frac{\pi r^2 h}{3}$; h

4. $I = p\,r\,t$; t

5. $A = \frac{h(b_1 + b_2)}{2}$; b_2

6. $F = \frac{9}{5}C + 32$

7. $\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$; q

8. $C = \frac{nE}{R + nr}$; n

9. $R = \frac{fs}{f + s}$; s

10. $S = \frac{rL - a}{r - 1}$; r

SECTION 3 INEQUALITIES

Given any two real numbers a and b, one of the following relations holds:

$a = b$; $a < b$; or $a > b$.

To solve an inequality, proceed much as you would to solve an equality. The one major exception is that multiplication of both sides of an inequality by a negative reverses the inequality sign. For example: $6 < 8$. If both sides are multiplied by -2 we have $-12 > -16$.

When working with inequalities keep in mind the following basic theorems for real numbers:

1. $a > b \iff a + c > b + c$

2. $a > b$ and $c > 0 \iff ac > bc$

3. $a > b$ and $c < 0 \iff ac < bc$

4. $a > b > c \Rightarrow a > c$

5. $a > b > c \iff a + d > b + d > c + d$

6. $a > b > c$ and $d > 0 \iff ad > bd > cd$

7. $a > b > c$ and $d < 0 \iff ad < bd < cd$

8. $a > b \iff \frac{1}{a} < \frac{1}{b}$ for $ab \neq 0$; $a > 0$; $b > 0$

9. $a > b > c \iff \frac{1}{a} < \frac{1}{b} < \frac{1}{c}$ for $abc \neq 0$; $a > 0$; $b > 0$; $c > 0$

10. $a^2 \geq 0$

11. $\sqrt{a^2} = |a|$

12. $|a| \geq 0$

13. $|a| = a$ if $a \geq 0$

14. $|a| = -a$ if $a < 0$

15. $|a| < b$ and $b > 0 \iff -b < a < b$

16. $|a| > b$ and $b > 0 \iff a > b$ or $a < -b$

17. $|a| = |b| \Rightarrow a = b$ or $a = -b$

18. $|a| \geq |b| \iff a^2 \geq b^2$

19. $|a| < |b| \iff a^2 < b^2$

Study the above theorems carefully. Possibly check them out with number examples.

Some examples: Solve each for x:

1. $4x - 3 < 2x + 8$

$2x - 3 < 8$ Th. 1

$2x < 11$ Th. 1

$x < 5.5$ Th. 2

2. $-10 < 4x + 4 < 2$

$-14 < 4x < -2$ Th. 5

$-\frac{14}{4} < x < -\frac{2}{4}$ Th. 6

$-\frac{7}{2} < x < -\frac{1}{2}$ Algebra

3. $|x + 2| < |x - 3|$

$(x + 2)^2 < (x - 3)^2$	Th. 19
$x^2 + 4x + 4 < x^2 - 6x + 9$	Algebra
$4x + 4 < -6x + 9$	Th. 1
$10x < 5$	Th. 1
$x < \frac{1}{2}$	Th. 2

4. $x^2 + 2x < 5$

$x^2 + 2x + 1 < 6$	Th. 1
$(x + 1)^2 < 6$	Algebra
$\|x + 1\| < \sqrt{6}$	Th. 11
$-\sqrt{6} < x + 1 < \sqrt{6}$	Th. 15
$-\sqrt{6} - 1 < x < \sqrt{6} - 1$	Th. 5

---

5. $x^2 + 2x < -1$

$x^2 + 2x + 1 < 0$	Th. 1
$(x + 1)^2 < 0$	Algebra
$\|x + 1\| < 0$	Th. 11
No solution	Th. 12

6. $x^2 + 2x > -1$

$x^2 + 2x + 1 > 0$	Th. 1
$(x + 1)^2 > 0$	Algebra
$\|x + 1\| > 0$	Th. 11
True for all $x \neq 1$	Th. 13

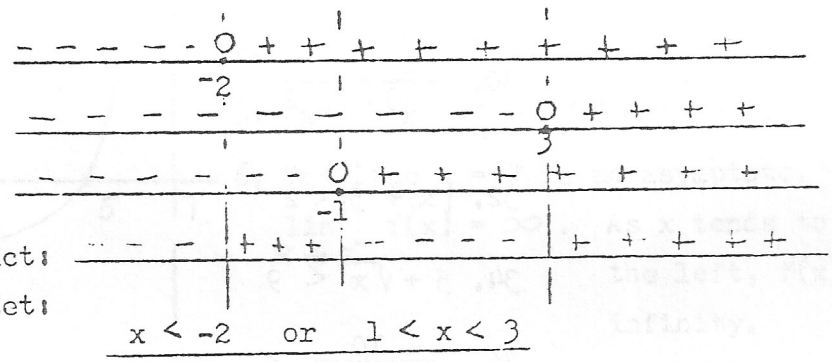
---

7. $(x + 2)(x - 3)(x + 1) < 0$

This example is somewhat different from the examples discussed above. Notice that we are asking for the product of three numbers to be negative. That will occur when an odd number of the factors is negative. The solution can be obtained graphically.

Study the graph of the signs of the three factors:

$(x + 2)$ (For $x = -2$, the factor is 0. For $x > -2$ the sign of the factor is positive. For $x < -2$, the sign of the factor is negative.)



Hence, the solution set:

$x < -2$ or $1 < x < 3$

EXERCISE 2 Solve each of the following inequalities for x:

1. $3x + 2 < x + 5$

2. $1 - 2x < 5x - 2$

3. $\frac{3x - 4}{5} \leq 0$

4. $x - 1 < 2x + 2 < x + 3$

5. $\frac{1}{2x - 5} > 0$

6. $\frac{1}{x + 4} < 0$

7. $|1 + 2x| \leq 1$

8. $|1 - 2x| \leq 1$

9. $|x - 3| > 5$

10. $|3x| > 9$

11. $|x - 2| \geq 8$

12. $|x - 1| > 5$

13. $|x - 3| < 2|x + 5|$

14. $|x^2| - 1 > 10$

15. $\frac{(x - 2)(x + 3)(x - 7)}{x + 1} \leq 0$

16. $x(x - 3)(x + 4) > 0$

17. $(x + 5)(x - 3)(x^2 - 4) \geq 0$

18. $4 < x^2 < 25$

19. $\frac{2x - 3}{x + 2} < \frac{1}{3}$

20. $3(x + 2)(2x - 3) < (x + 2)^2$

21. $x - 4 < \frac{2x}{3} + \frac{2 - 3x}{5}$

22. $3x < 2x^2$

23. $x^2 > x$

24. $|3x + 2| < 7$

25. $(x + 2)(x - 3)(x + 8) \geq 0$

26. $(2 - x)(3 + x)(5 - x) \leq 0$

27. $|x^2 - 4| < 5$

28. $\left|\frac{x - 2}{x + 1}\right| > 0$

29. $\left|\frac{7 - 3x}{2}\right| \leq 1$

30. $\frac{2}{x^3} < \frac{1}{x^2}$

31. $-1 < \frac{3 - 5x}{4} \leq 6$

32. $|x + 5| < 2$

33. $x < |x|$

34. $5 + \sqrt{x} < 9$

35. $\frac{-x}{x^2 - 2x - 15} \geq 0$

36. $\frac{10}{x^2 + 16} < 0$

SECTION 4.6

GRAPHING FUNCTIONS

This section incorporates techniques of solving equations, solving inequalities, finding limits, and plotting points, in order to sketch a rough graph of a function. In earlier work it was important to be able to identify the general shape of a graph of certain functions. See if you remember:

Match the following--check it out with a friend.

1. $f(x) = 2x + 3$	A. Parabola
2. $f(x) = x^2$	B. V shape
3. $f(x) = (x - 3)^2 + 2$	C. Line
4. $f(x) = \frac{1}{x}$	D. Hyperbola
5. $f(x) = \lvert x \rvert$	

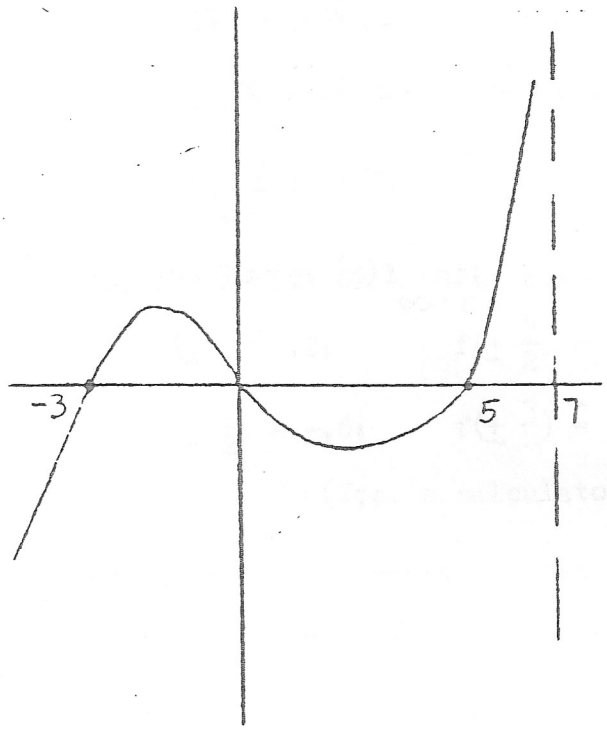
The <u>domain</u> of a function $f(x)$ is the set of real numbers x for which the function is defined.

The <u>range</u> of a function $f(x)$ is the set of real numbers which are the values of $f(x)$.

An <u>even function</u> is a function $f(x)$ such that $f(x) = f(-x)$. The graph of an even function has Y-axis symmetry.

An <u>odd function</u> is a function $f(x)$ such that $f(-x) = -f(x)$. The graph of an odd function has origin symmetry.

Study the grpah below:

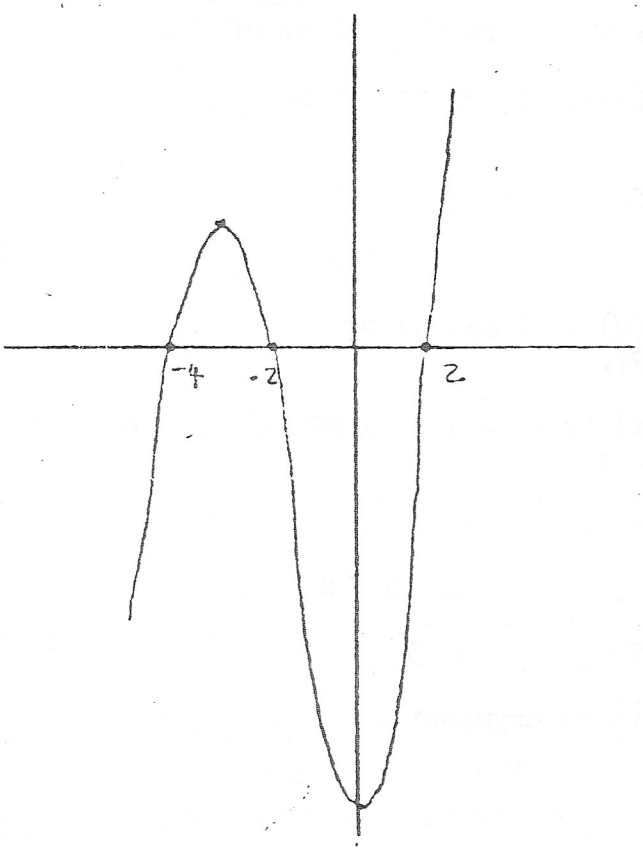


1. The domain of $f(x)$ is $(-\infty, 7)$ or $\{x: x < 7\}$.
2. The range of $f(x)$ is $(-\infty, \infty)$ or all real numbers.
3. $f(x) = 0$ for $x = -3, 0, 5$.
4. $f(x) > 0$ for $-3 < x < 0$ or $5 < x < 7$
5. $f(x) < 0$ for $0 < x < 5$ or $x < -3$
6. The line $x = 7$ is an asymptote. $\lim_{x \to 7^-} f(x) = \infty$. As x tends to 7 from the left, $f(x)$ tends to infinity.
7. This function is neither even nor odd.
8. The Y-intercept of the function is 0.

To graph a function it is useful at times to:

1. Identify the domain of the function.
2. Identify the range of the function.
3. Locate any asymptotes that the graph might have.
4. Determine whether the function is odd, even, or neither.
5. Locate the zeros of the function. That is, find the values of x for which $f(x) = 0$. These values of x are the X-intercepts.
6. Determine those values of x for which $f(x) = 0$. These values of x are the Y-intercepts.
7. Determine x such that $f(x) > 0$.
8. Determine x such that $f(x) < 0$.
9. Plot any significant points that you feel are necessary.

Example 1: Graph the function: $f(x) = (x - 2)(x + 2)(x + 4)$



1. The domain is all reals.
2. The range is all reals.
3. The function is neither even nor odd.
4. The zeros of the function are:
   2, -2, and -4
5. $- - - - - 0 + + + + + + + + +$ (at -2)
   $- - - - - - - - 0 + + + + +$ (at 2)
   $- 0 + + + + + + + + + + +$ (at -4)

   $f(x) > 0$ for $x > 2$ or $-4 < x < -2$

   $f(x) < 0$ for $x < -4$ or $-2 < x < 2$
6. $f(-3) = 5$

   $f(0) = -16$
7. $\lim_{x \to \infty} f(x) = \infty$; $\lim_{x \to -\infty} f(x) = -\infty$

Example 2: $f(x) = \dfrac{1}{2x^2 - 2}$

1. Since $2x^2 - 2$ is a denominator, $2x^2 - 2 \neq 0$. Hence $x^2 \neq 1$; $|x| \neq 1$
   The domain is all real numbers x such that $x \neq \pm 1$.

2. $\lim\limits_{x \to 1^+} \dfrac{1}{2x^2 - 2} = \infty$. As x is just a bit more than 1, $2x^2 - 2$ is a a very small positive number.

   Likewise; $\lim\limits_{x \to -1^-} f(x) = \infty$.

   The lines $x = 1$ and $x = -1$ are asymptotes for f(x).

3. $\lim\limits_{x \to 1^-} f(x) = -\infty$. Likewise, $\lim\limits_{x \to -1^+} f(x) = -\infty$.

4. The function is even. Notice, $f(x) = f(-x)$. The graph will have Y-axis symmetry.

5. f(x) will never equal zero. There are no X-intercepts.

6. f(x) will be greater than zero as long as $2x^2 - 2 > 0$.

   $2x^2 > 2$; $x^2 > 1$; $|x| > 1$; $\underline{x > 1 \text{ or } x < -1.}$

7. f(x) will be less than zero as long as $2x^2 - 2 < 0$.

   $2x^2 < 2$; $x^2 < 1$; $|x| < 1$; $\underline{-1 < x < 1}$

8. $f(0) = -\dfrac{1}{2}$

9. $\lim\limits_{x \to +\infty} f(x) = 0$; $\lim\limits_{x \to -\infty} f(x) = 0$

10. The X-axis is a horizontal asymptote.

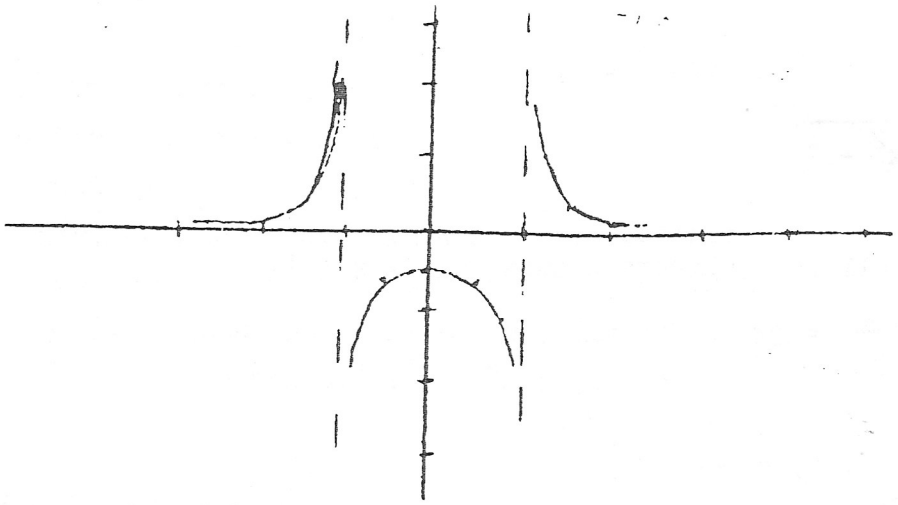
11. $f(\pm 2) \doteq .2$; $f(\pm \frac{3}{2}) = .4$; $f(\pm \frac{6}{5}) \doteq 1.1$

    $f(\pm \frac{1}{2}) \doteq -.6$; $f(\pm \frac{3}{4}) \doteq -1.1$

    (Yes, a calculator is handy!)

Study the above information. Try your hand at sketching the graph of this function. By the way, what is the range of the funotion?

THE GRAPH IS ON THE NEXT PAGE. SKETCH THE GRAPH AND THEN TURN THE PAGE TO SEE IF YOUR SKETCH MATCHES THAT OF THE MANAGEMENT!



EXERCISE 3 Graph as many of the following functions as you need in order to feel like quite an expert. Be prepared to answer questions regarding:

1. Domain; 2. Range; 3. Equations of asymptotes; 4. X-intercepts;
5. Y-intercepts; 6. Whether the function is even, odd, or neither;
7. For what values of x the function is greater than 0;
8. For what values of x the function is less than 0.
9. The limit of the function as x tends to a given number.
10. The value of the function for a given domain element.

1. $f(x) = (x - 4)(x + 1)(x + 2)$

2. $f(x) = x(x + 3)$

3. $f(x) = \dfrac{x + 1}{x - 2}$

4. $f(x) = \dfrac{(x + 1)(x - 2)}{x}$

5. $f(x) = x^2 + 4x + 3$ (Factor)

6. $f(x) = x^2 - 4$

7. $f(x) = \left| \dfrac{3}{x + 2} \right|$

8. $f(x) = x^3 - 3x$

9. $f(x) = \left| x^4 - x^2 \right|$

10. $f(x) = \dfrac{x - 2}{x - 1}$

11. $f(x) = \dfrac{12}{4 + x^2}$

12. $f(x) = \dfrac{x(x + 1)}{x - 2}$

13. $f(x) = \dfrac{x - 3}{(x + 2)(x + 1)}$

14. $f(x) = x^5 - 16x$

15. $f(x) = \sqrt{x - 4}$

16. $f(x) = \sqrt{4 - x^2}$

17. $f(x) = \log_2 |x|$

18. $f(x) = \left| \log_2 x \right|$

19. $f(x) = 2^x$

20. $f(x) = \left| \sin x \right|$

21. $f(x) = [\![\sin x]\!]$

22. $f(x) = 2^{-x}$

23. $f(x) = -[\![x]\!]$

24. $f(x) = 2^{\frac{1}{x}}$

25. $f(x) = \begin{cases} 2x \text{ for } x \geq 1 \\ x^2 \text{ for } x < 1 \end{cases}$

26. $f(x) = \begin{cases} [\![x]\!] \text{ for } x \geq 0 \\ |x| \text{ for } x < 0 \end{cases}$

27. $f(x) = \left| \frac{1}{\sin x} \right|$

28. $f(x) = 2 \left| [\![\sin x]\!] \right|$

29. $f(x) = [\![\log_2 x]\!]$ for $x \geq 1$

30. $f(x) = \left| \frac{x^3}{2 - x} \right|$

MATH ANALYSIS ONLY

31. $f(x) = (x - a_1)(x - a_2)(x - a_3)$ for $a_1 \neq a_2 \neq a_3$. What are the zeros of this function?

$\lim_{x \to +\infty} f(x) =$ ____; $\lim_{x \to -\infty} f(x) =$ ____.

32. $f(x) = k(x - a_1)(x - a_2)$ for $a_1 \neq a_2$,

For $k > 0$, $\lim_{x \to +\infty} f(x) =$ ____; $\lim_{x \to -\infty} f(x) =$ ____

For $k < 0$; $\lim_{x \to +\infty} f(x) =$ ____; $\lim_{x \to -\infty} f(x) =$ ____.

THE END. TAKE THE TRIAL RUN. TAKE THE TEST.

ANSWERS

EXERCISE 1

1. No solution
2. 5.5
3. $-\frac{3}{5} \leq x < -\frac{2}{5}$
4. $x = -4$, $x = -1$;
5. 1;
6. $\frac{3}{7}$;
7. $\frac{-7 \pm \sqrt{97}}{2}$
8. No solution;
9. $-\frac{21}{8}$;
10. 2;
11. 69
12. $-\frac{21}{2}$; $-\frac{9}{2}$;
13. 12;
14. -4;
15. 3;
16. -3;
17. 2;
18. 25
19. $\frac{-1 \pm \sqrt{13}}{3}$;
20. -12;
21. 21;
22. 0; 6;
23. ± 4;
24. $-\frac{8}{9}$;
25. 0;
26. 19

ANSWERS Cont.

27. $5\sqrt{2}$; 28. $\frac{5}{2}$; 29. 1; 30. 6; 31. 0, $\frac{1}{4}$, -2

32. $\frac{3}{2}$; 1; 33. 1; 34. 19; 35. 70; 36. 3;

37. -1, -2; 38. $\frac{\log 4}{\log 5}$; 39. $\frac{5}{2}$; 40. 6, 2

EXERCISE 2

1. $\frac{p - 2L}{2}$; 2. $\frac{2S_n}{n} - a_1$; 3. $\frac{3V}{\pi r^2}$; 4. $\frac{I}{pr}$; 5. $\frac{2A}{h} - b_1$;

6. $\frac{5(F - 32)}{9}$; 7. $\frac{fp}{p - f}$; 8. $\frac{CR}{E - Cr}$; 9. $\frac{Rg}{g - R}$; 10. $\frac{S - a}{S - L}$

EXERCISE 3

1. $x < \frac{3}{2}$; 2. $x > \frac{3}{7}$; 3. $x \leq \frac{4}{3}$; 4. $-3 < x < 1$;

5. $x > \frac{5}{2}$; 6. $x < -4$; 7. $-1 \leq x \leq 0$; 8. $0 \leq x \leq 1$

9. $x > 2$ or $x < -8$; 10. $x > 3$ or $x < -3$; 11. $x \geq 6$ or $x \leq -10$;

12. $x > 6$ or $x < -4$; 13. $x < -13$ or $x > -\frac{7}{3}$; 14. $x > \sqrt{11}$ or $x < -\sqrt{11}$

15. $-3 \leq x < -1$ or $2 \leq x \leq 7$; 16. $x > 3$ or $-4 < x < 0$;

17. $x \leq -5$ or $-2 \leq x \leq 2$ or $x \geq 3$; 18. $-5 < x < -2$ or $2 < x < 5$;

19. $-2 < x < \frac{11}{5}$; 20. $-2 < x < \frac{11}{5}$; 21. $x < \frac{33}{7}$;

22. $x < 0$ or $x > \frac{3}{2}$; 23. $x > 1$ or $x < 0$ 24. No solution.

25. $-8 \leq x \leq -2$ or $x \geq 3$; 26. $x \leq -3$ or $2 \leq x \leq 5$; 27. $-3 < x < 3$;

28. $x \neq -1$, $x \neq 2$; 29. $\frac{5}{3} \leq x \leq 3$; 30. $x < 0$ or $x > 2$;

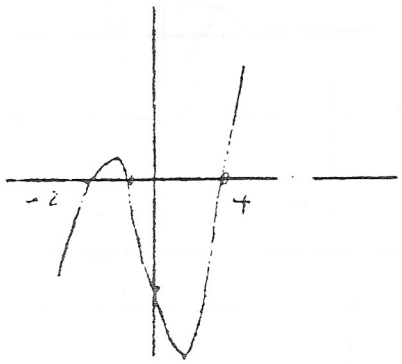
31. $-\frac{21}{5} \leq x < \frac{7}{5}$; 32. $-7 < x < -3$; 33. $x < 0$;

34. $0 \leq x < 16$; 35. $x < -3$ or $0 \leq x < 5$; 36. No solution.

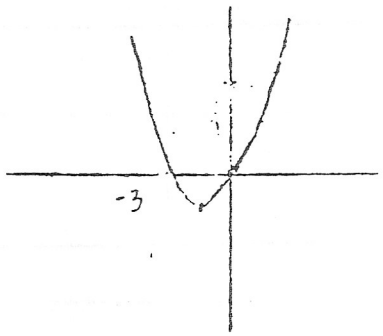
Answers Cont.

EXERCISE 4 Below are rugh sketches of the graphs.

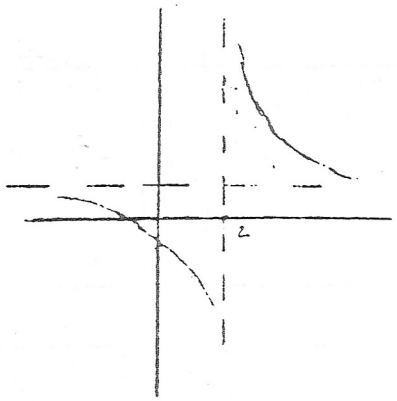
1.



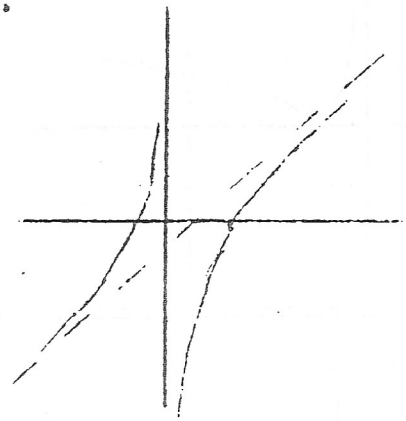
2.



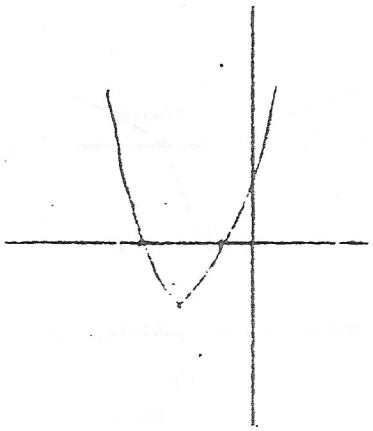
3.



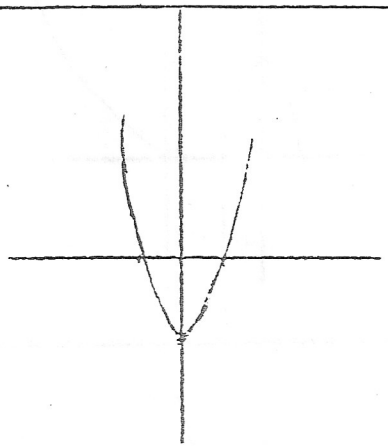
4.



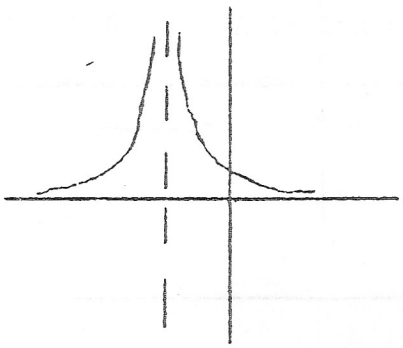
5.



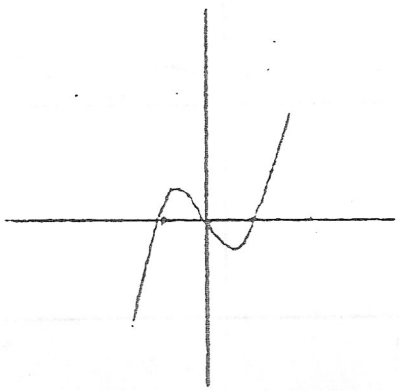
6.



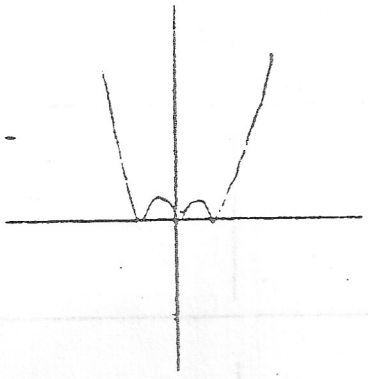
7.



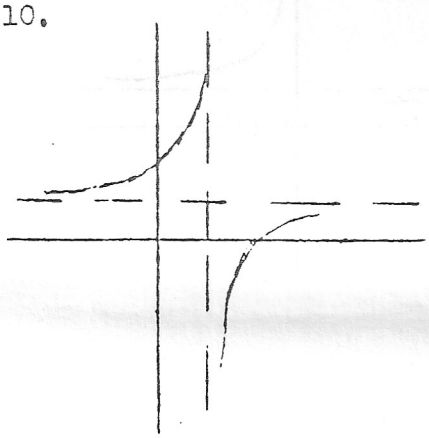
8.



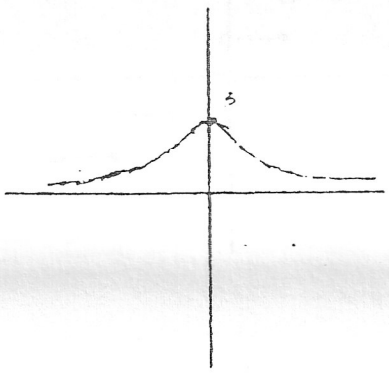
9.



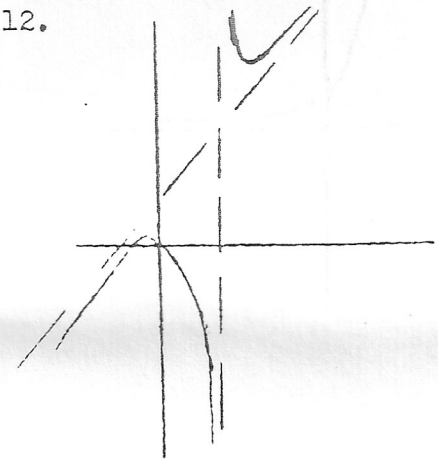
10.



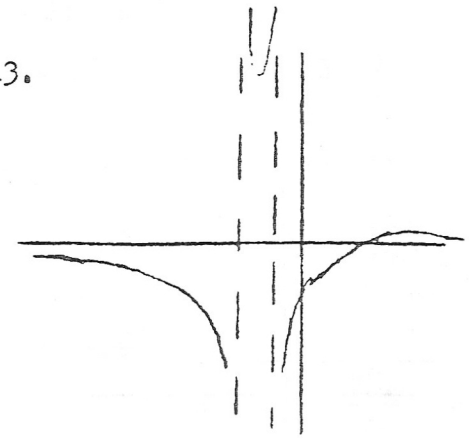
11.



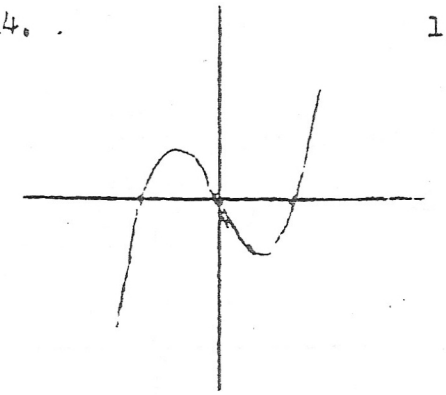
12.



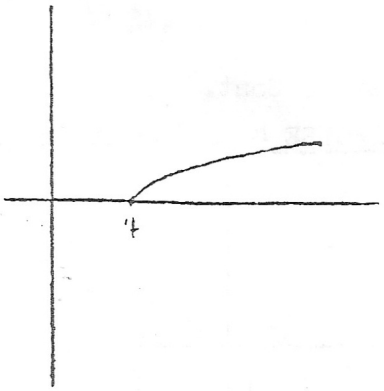
13.



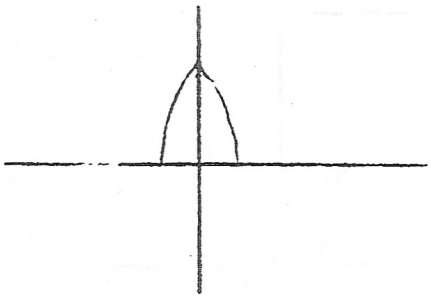
14.



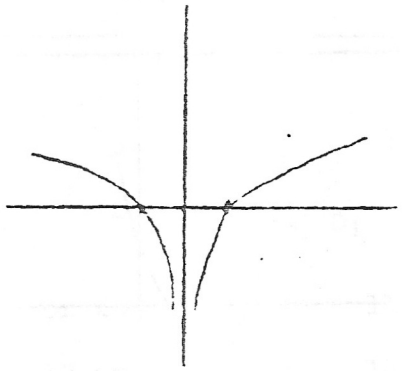
15.



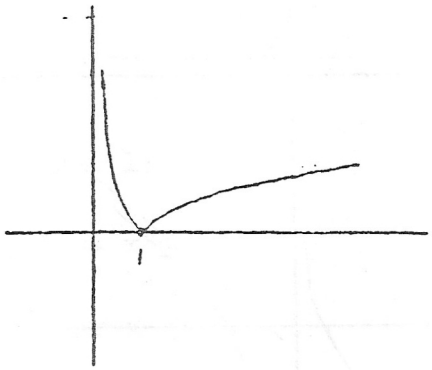
16.



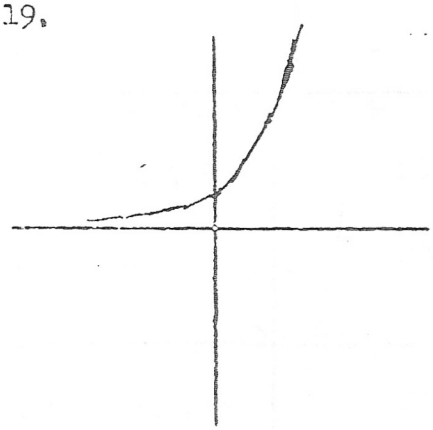
17.



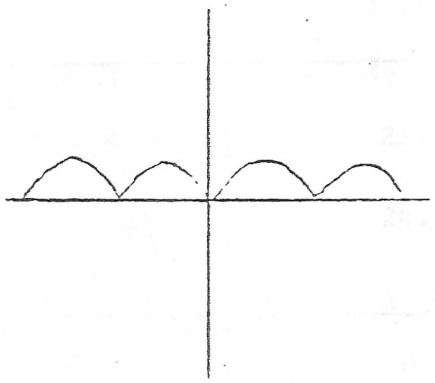
18.



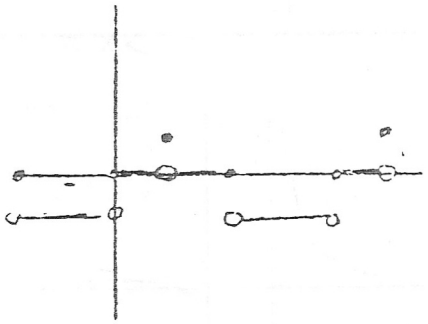
19.



20.

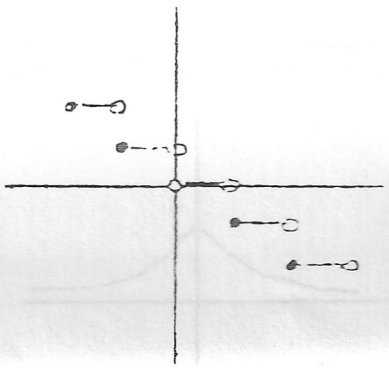


21.

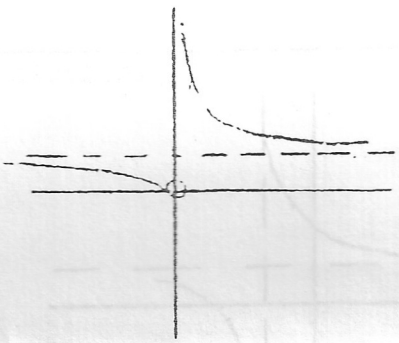


22.

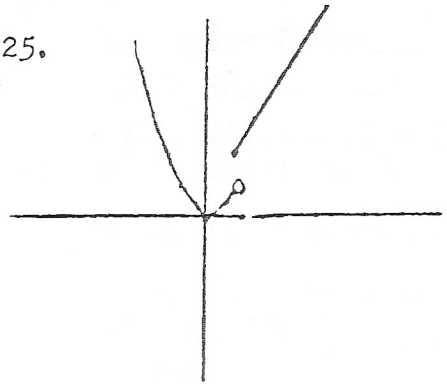
23.



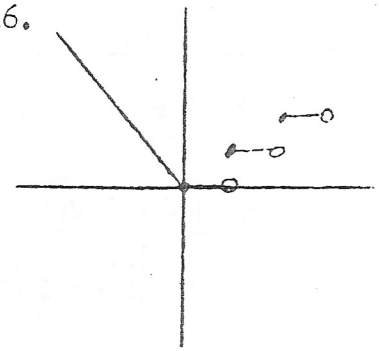
24.



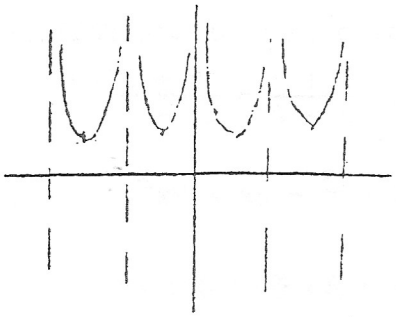
25.



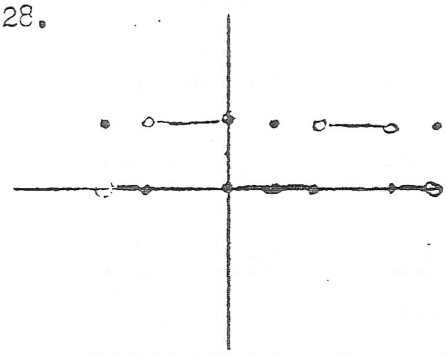
26.



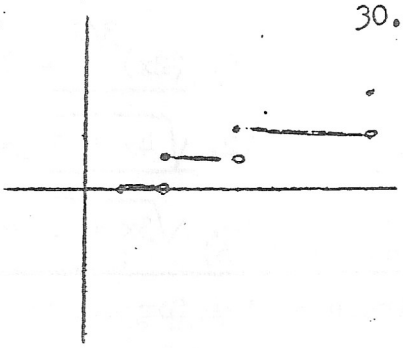
27.



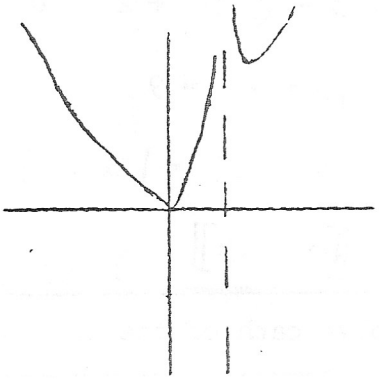
28.



29.



30.



31. The zeros are a_1, a_2, and a_3. $\lim_{x \to \infty} f(x) = \infty$.; $\lim_{x \to -\infty} f(x) = -\infty$

32. For $k > 0$ both limits are ∞

For $k < 0$, both limits are $-\infty$.

EQUATIONS AND FUNCTIONS

TRIAL RUN

I. Solve each of the following for all real numbers x:

1. $\frac{x+3}{x-2} = \frac{x+2}{2x-1}$

2. $(3x+1)(2x+5) = (1-6x)(4-x)$

3. $4x^3 = 2x - 7x^2$

4. $[\![x + 1,3]\!] = 4$

5. $\log_3 x^2 = 4$

6. $5x^{3/2} = 135$

7. $5 - \sqrt[3]{3x^2 + 2} = 0$

8. $4x^4 - 39x^2 + 27 = 0$

9. $|3 + 4x| = 9$

10. $(2x)^{3/2} = 8$

11. $|5x - 2| = |2x + 1|$

12. $\sqrt{4x + 9} = 7$

13. $[\![x + .3]\!] = x$

14. $\sqrt{3x + 1} = -8$

II. Solve each of the following inequalities for all real numbers x:

1. $x^2 \leq 13x$

2. $\frac{1}{x^2} < 64$

3. $\frac{-x}{x^2 - x - 12} \geq 0$

4. $|x^2 - 4| < 3$

5. $\frac{(x+2)(x+3)}{x-1} \geq 0$

6. $|x - 3| \leq |x + 2|$

7. $|3x + 5| > 2$

8. $5 \leq 3x + 2 \leq 17$

9. $-2 \leq \frac{2x+3}{5} \leq 2$

10. $\frac{1}{2x-5} > 0$

11. $\log_2 x \leq 3$

12. $\log_2 |x| \leq 3$

III. Graphs of functions:

1. For the function, $f(x) = \sqrt{9 - x^2}$, (a) The domain is _____

   (b) The range is _____

2. For the function, $f(x) = \frac{(x+2)(x+5)}{x(x-3)}$

   (a) Vertical asymptotes for the graph are the lines: _____

   (b) $f(x) > 0$ for _____

   (c) $f(x) < 0$ for _____

   (d) $f(x) = 0$ for _____

TRIAL RUN CONTINUED

3. For the function $f(x) = (x^2 - 1)(x^2 - 2)$

(a) $f(x) = 0$ for _____; (b) $f(0) =$ _____

(c) Is the function even, odd, or neither? _____

4. For the function, $f(x) = \frac{(x + 2)(x + 1)}{(3x - 1)(x + 3)}$

(a) Vertical asymptotes are the lines: _____

(b) $f(x) > 0$ for _____

(c) $\lim_{x \to -3^+} f(x) =$ _____ (The limit as x tends to -3 from the right.)

*5. $f(x) = k(x + 2)(x - 3)(x + 8)$

(a) For $k < 0$, $\lim_{x \to \infty} f(x)$ is _____

(b) For $k > 0$ $\lim_{x \to -\infty} f(x) =$ _____

(c) The zeros of $f(x)$ are _____; (d) The domain of $f(x)$ is _____

6. Review the graphins exercises of the L.A.P.

ANSWERS

I. 1. $\frac{-5 \pm \sqrt{21}}{2}$; 2. $-\frac{1}{42}$; 3. 0, -2, $\frac{1}{4}$; 4. $2.7 \leq x < 3.7$

5. ± 9; 6. 9; 7. $\pm \sqrt{41}$; 8. ± 3; $\pm \frac{\sqrt{3}}{2}$; 9. $\frac{3}{2}$, -3;

10. 2; 11. 5, $-\frac{11}{7}$; 12. 10; 13. All integers x; 14. No solution.

II. 1. $0 \leq x \leq 13$; 2. $x > \frac{1}{8}$ or $x < -\frac{1}{8}$; 3. $x < -3$ or $0 \leq x < 4$;

4. $-\sqrt{7} < x < -1$ or $1 < x < \sqrt{7}$; 5. $-3 \leq x \leq -2$ or $x > 1$; 6. $x \geq \frac{1}{2}$;

7. $x > -1$ or $x < -\frac{7}{3}$; 8. $1 \leq x \leq 5$; 9. $-\frac{13}{2} \leq x \leq \frac{7}{2}$; 10. $x > \frac{5}{2}$;

11. $0 < x \leq 8$; 12. $-8 \leq x < 0$ or $0 < x \leq 8$

III. 1. (a) $-3 \leq x \leq 3$; (b) $0 \leq y \leq 3$

2. (a) $x = 0$ and $x = 3$; (b) $x < -5$ or $-2 < x < 0$ or $x > 3$

(c) $-5 < x < -2$ or $0 < x < 3$; (d) $x = -2$ or $x = -5$

3. (a) $x = \pm 1$ or $x = \pm \sqrt{2}$; (b) 2; (c) even.

4. (a) $x = \frac{1}{3}$ or $x = -3$; (b) $-3 > x$ or $-2 < x < -1$ or $x > \frac{1}{3}$;

(c) $-\infty$.

5. (a) $-\infty$; (b) $-\infty$; (c) -2, 3, -8; (d) All real numbers.