MATH ANALYSIS   GRAPHING L.A.P. 1

TRIGONOMETRY

BEHAVIORAL OBJECTIVES

I. Define

A. A periodic function
B. The period of a function
C. The amplitude of
   1. $f(x) = a \sin x$
   2. $f(x) = a \cos x$

II. For $f(x) = a \sin(x + k)$

A. Graph the function

B. Determine the

1. Domain
2. Range
3. Amplitude
4. Phase shift
5. Zeros

III. Repeat II for (For B-E., omit amplitude; include equations of asymptotes.)

A. $f(x) = a \cos (x + k)$
B. $f(x) = a \tan (x + k)$
C. $f(x) = a \csc (x + k)$
D. $f(x) = a \sec (x + k)$
E. $f(x) = a \cot (x + k)$

* IV. Graph, determine domain, range, zeros, and asymptotes for the trig functions when

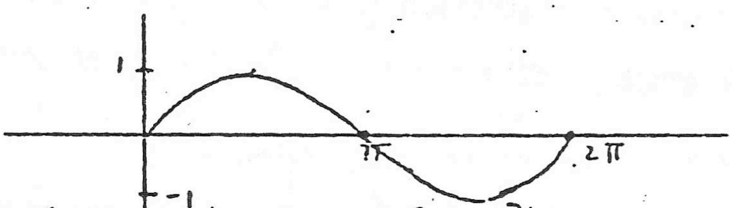
A. Absolute value is involved
B. Greatest integer is involved

*V. Graph a compound function using the addition of ordinates method.

SECTION I   VARIATIONS ON THE SIN CURVE

For the function: $f(x) = \sin x$

The graph of one period is:



$f(x) = \sin x$ is a function with domain $(-\infty, \infty)$ and range $[-1, 1]$

The amplitude of the curve is 1. The amplitude of a sin curve is $\frac{1}{2}$ the difference between the maximum y value and the minimum y value.

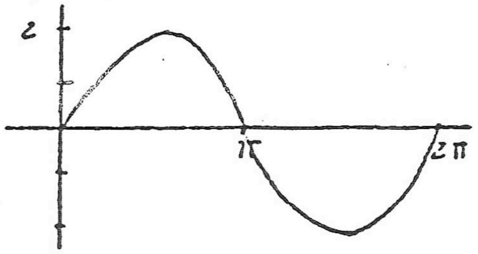
The period of $f(x) = \sin x$ is 2π.

Throughout this section the primary graph shown above will be the reference graph. Although we focus on only one period be aware that the domain is $(-\infty, \infty)$ and the curve continually repeats at intervals of 2π.

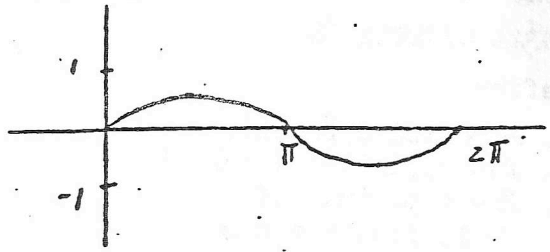
Turn the page and we shall mess up the sin curve!

A. The T.S. (Taller - Shorter) Factor

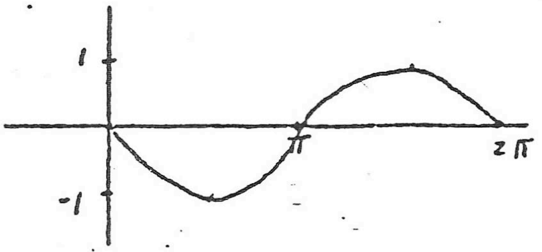
$f(x) = 2 \sin x$



$f(x) = \frac{1}{2} \sin x$



$f(x) = - \sin x$



In general: $f(x) = a \sin x$. "a" affects the amplitude. The amplitude of $f(x) = a \sin x$ is $|a|$.

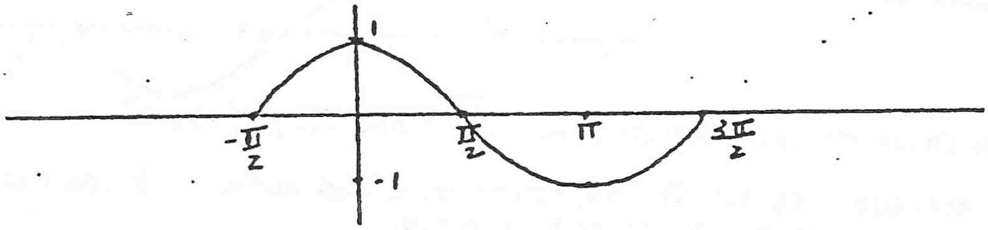
If $|a| > 1$ the curve is "taller" than the reference graph.
If $|a| < 1$ the curve is "shorter" than the reference graph.
A negative "a" causes the reference curve to be reflected about the X-axis.

B. The L.R. (Left-Right) Shifter

Consider the function: $f(x) = \sin\left(x + \frac{\pi}{2}\right)$. Our reference curve begins with $f(x) = \sin 0$, and ends with $f(x) = \sin 2\pi$. Hence, to get the beginning of this changed curve we ask: For what value of x does $x + \frac{\pi}{2} = 0$? And then we ask: For what value of x does $x + \frac{\pi}{2} = 2\pi$?

The answer to question #1 is $x = \frac{-\pi}{2}$. The answer to question #2 is $x = \frac{3\pi}{2}$.

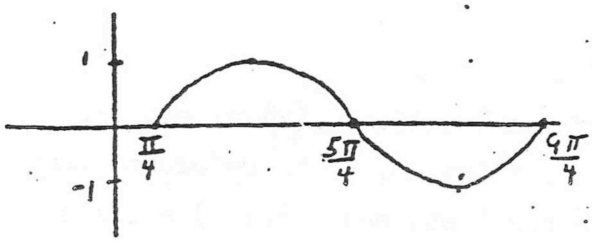
The graph of $f(x) = \sin\left(x + \frac{\pi}{2}\right)$ looks like this:



The curve has a phase shift of $\frac{\pi}{2}$ units to the left.

Before you turn the page, graph $f(x) = \sin\left(x - \frac{\pi}{4}\right)$.

Here is the graph of $f(x) = \sin(x - \frac{\pi}{4})$



In this graph the curve shifts $\frac{\pi}{4}$ units to the right.

In general: $f(x) = \sin(x + c)$

c is a left-right shifter.

If $c > 0$, the curve shifts $|c|$ units to the left.

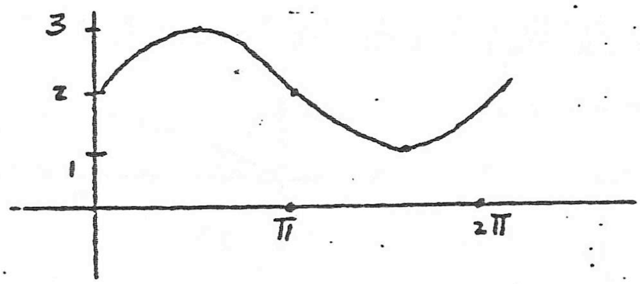
If $c < 0$, the curve shifts $|c|$ units to the right.

These $|c|$ units are sometimes called the phase shift, or the translation.

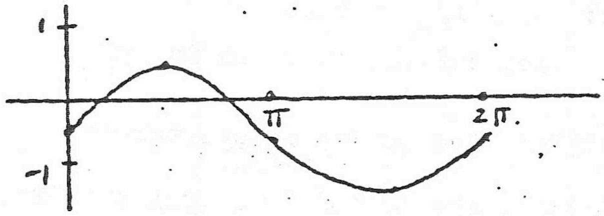
C. The U. D. (Up-Down) Shifter

Take a look at the function $f(x) = \sin x + 2$. For this function, each ordinate of the regular sin curve is merely increased by 2 units. Hence the curve is raised 2 units.

See: $f(x) = \sin x + 2$



How about: $f(x) = \sin x - \frac{1}{2}$



In general for $f(x) = \sin x + k$

If $k > 0$ ~~the graph~~ shifts up $|k|$ units.

If $k < 0$ the graph shifts down $|k|$ units.

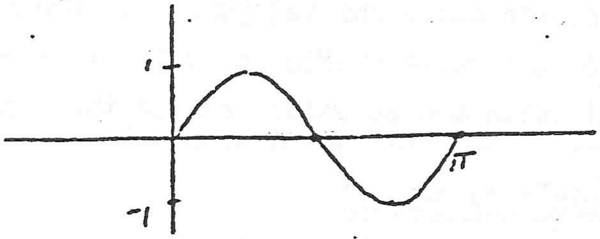
D. The S.S. (Stretcher-Shrinker) factor

A function is said to be periodic if $f(x + P) = f(x)$ for all values of x, where P is a constant. The period of the function is the smallest positive value of P for which the relation is true. For the sin function the period is 2π. Note: $\sin x = \sin(x + 2\pi)$. And 2π is the smallest positive number for which this is true. $f(x) = \sin x$; $f(x) = \cos x$; $f(x) = \sec x$; and $f(x) = \csc x$ each have a period of 2π. $f(x) = \tan x$ and $f(x) = \cot x$ have a period of π.

Consider the function $f(x) = \sin 2x$.

Again we think about the reference graph for $f(x) = \sin x$. It begins at $f(x) = \sin 0$ and ends at $f(x) = \sin 2\pi$. Now we ask: For what value of x does $2x = 0$? and for what value of x does $2x = 2\pi$. The answer to the first question is $x = 0$. The answer to the second question is $x = \pi$. Thus the new primary curve begins at $x = 0$ and ends at $x = \pi$. The function $f(x) = \sin 2x$ has a period of π.

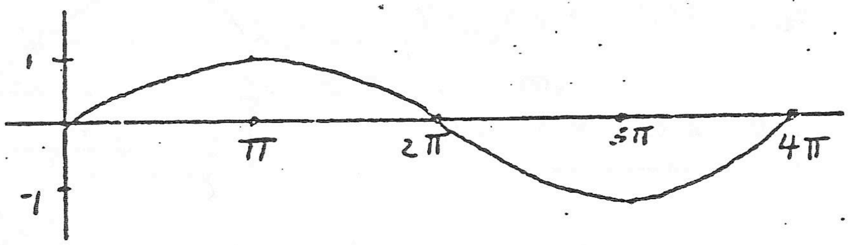
Here is the graph:



Another example: $f(x) = \sin \frac{1}{2} x$

(1) $\frac{1}{2} x = 0$ for $x = 0$; (2) $\frac{1}{2} x = 2\pi$ for $x = 4\pi$.

Here is the graph: Note: the period is 4π.



In general: for $f(x) = \sin bx$

The period of the function is $\frac{2\pi}{|b|}$

If $|b| > 1$, the period is shorter than 2π.

If $|b| < 1$, the period is longer than 2π.

E. ALL TOGETHER

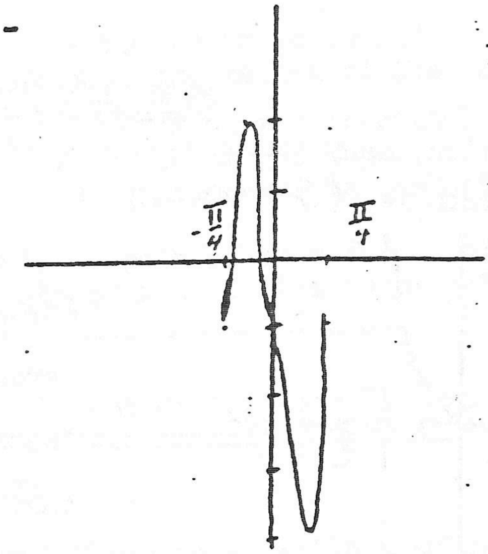
Suppose we have the function $f(x) = 3 \sin(4x + \pi) - 1$

By observing the function and remembering the previous business we observe the following:

1. The amplitude is 3. 2. The period is $\frac{\pi}{2}$.
3. The phase shift is left $\frac{\pi}{4}$ units.
4. The negative 1 moves the curve down 1 unit.

Now, graph the curve before looking at its graph on the next page.

Here it is: $f(x) = 3\sin(4x + \pi) - 1$



In general:

For $f(x) = a \sin(bx + c) + k$

Amplitude: a

Period: $\frac{2\pi}{b}$

Phase shift: $\frac{-c}{b}$

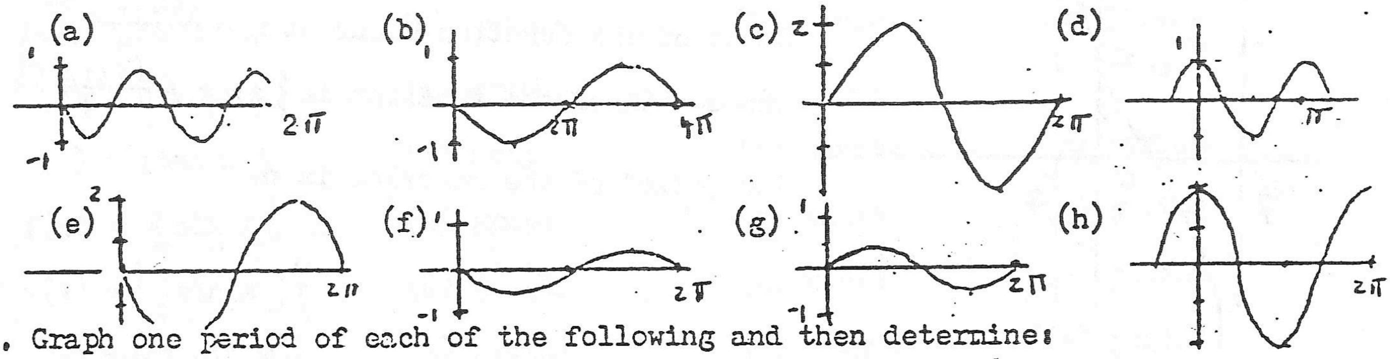
Raise-Lower: k

ASSIGNMENT 1

1. For each equation given determine its corresponding graph. Give amplitude and period for each.

   1. $y = 2 \sin x$ 2. $y = \frac{1}{2} \sin x$ 3. $y = -\sin 2x$ 4. $y = -\sin \frac{x}{2}$

   5. $y = -2 \sin x$ 6. $y = -\frac{1}{2} \sin x$ 7. $y = 2 \cos x$ 8. $y = \cos 2x$



2. Graph one period of each of the following and then determine:

   (a) Range; (b) Amplitude; (c) Period; (d) Phase shift

   A. $f(x) = \sin 3x$ B. $f(x) = 2 \sin \frac{x}{2}$ C. $y = -2 \sin x + 1$

   D. $y = 4 \sin(x - \frac{\pi}{4})$ E. $y = \cos x + 1$ F. $f(x) = 2 \cos 2x$

   G. $f(x) = 5 \cos(x + \pi)$ H. $y = -\cos 4x$ I. $y = -\sin(x + \frac{\pi}{2}) + 1$

3. The curve of $y = \sin x$ is shifted $\frac{\pi}{4}$ units to the right. What is the equation of the new curve?

4. The amplitude of $y = \cos x$ is multiplied by 7. What is the equation of the new curve.

5. The curve $y = \sin x$ is shifted π units to the right and the period of the curve is doubled. What is the equation of the new curve?

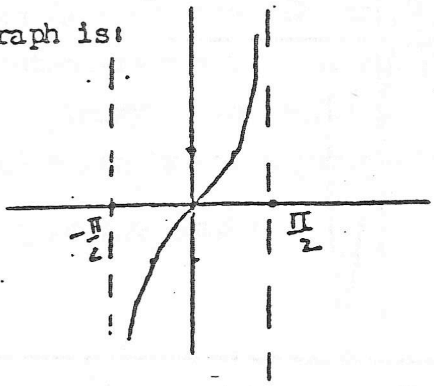
6. The curve $y = \sin x$ is reflected about the X-axis and then the entire curve is raised 3 units. What is the equation of the new curve?

SECTION II. MORE TRIG FUNCTION GRAPHS

Suppose we consider the function $f(x) = 2 \tan (x + \frac{\pi}{4})$

Our reference graph is $f(x) = \tan x$.

Its graph is:



Asymptotes occur at $x = \frac{(2k + 1)\pi}{2}$

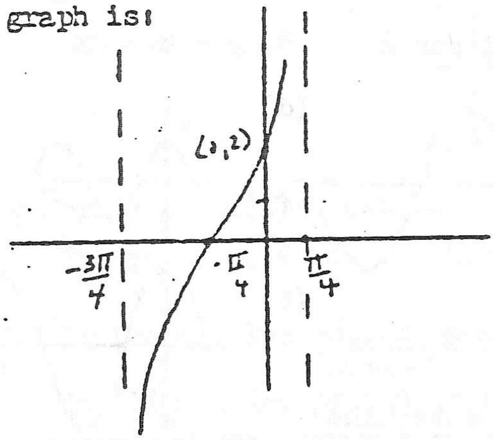
Zeros of the function occur at $x = k\pi$

The domain of the function is $\{x: x \neq \frac{(2k+1)\pi}{2}\}$

The period of the function is π.

For the function $f(x) = 2 \tan(x + \frac{\pi}{4})$, the phase shift is $\frac{-\pi}{4}$. The entire picture is shifted $\frac{\pi}{4}$ units to the left. The initial factor 2, causes each ordinate to be multiplied by 2. Hence, the new graph rises more rapidly than the graph of the reference curve.

The graph is:



Asymptotes occur at $x = \frac{(4k + 1)\pi}{4}$

Zeros of the function occur at $x = \frac{(4k \div 1)\pi}{4}$

The domain of the function is $\{x: x \neq \frac{(4k+1)\pi}{4}\}$

The period of the function is π.

In general: for ~~f(x)~~ $= a \tan (bx + c) + k$

Period: $\frac{\pi}{|b|}$ Phase shift: $\frac{-c}{b}$

Note: Amplitude is only considered relative to the sin and cos functions.

ASSIGNMENT 2 Do each of these carefully, very carefully. Graph at least one period.

1. Graph: $f(x) = 2 \tan 2x$ (a) What are the zeros; (b) What are the equations of the asymptotes

2. Graph: $f(x) = 2 \csc \frac{x}{2}$ (a) What is the range? (b) What is the domain?
(c) What are the equations of the asymptotes?
(d) What is the period?

3. Graph $f(x) = \frac{1}{4} \cot (x - \frac{\pi}{2}) + 1$
   (a) What is the phase shift?
   (b) What is the period of the function?

4. Graph: $f(x) = -2 \sec 4x$
   (a) What are the zeros of the function?
   (b) What is the period of the function?
   (c) What are the equations of the asymptotes?

5. Graph: $f(x) = - \tan (x + \pi)$
   (a) What is the domain of this function?
   (b) What is the period of this function?

Trig Students: Procede now to the Trial Run. Do not do Sections III and IV.

SECTION III. UNUSUAL GRAPHS

Now it is time to put lots of things together. Throw in a little absolute value, greatest integer, and inequalities. The results just might prove interesting.

Review: $f(x) = [\![x]\!]$ is read f(x) = the greatest integer which does not exceed x.

Examples: $[\![3.2]\!] = 3$; $[\![-4]\!] = -4$; $[\![-4.1]\!] = -5$; $[\![8.7]\!] = 8$

Try these:

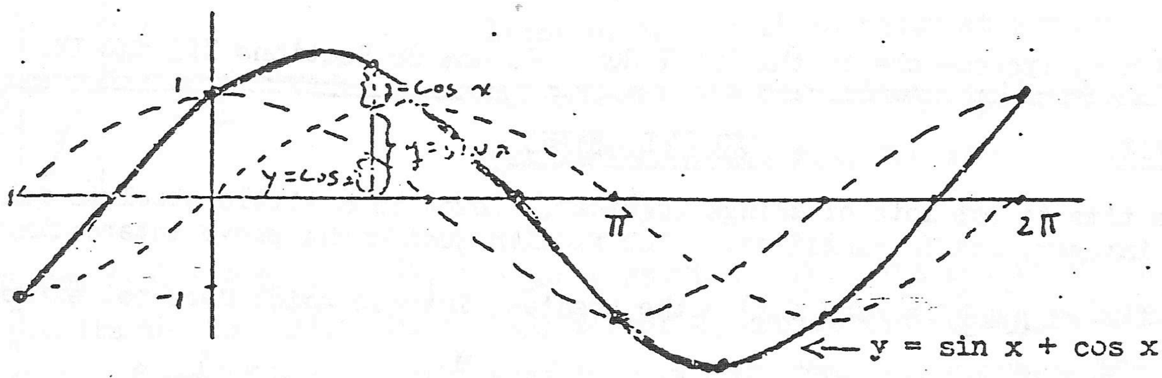
ASSIGNMENT 3

1. $f(x) = \lvert \sin x \rvert$	(a) graph;	(b) range;	(c) period
2. $f(x) = \sin \lvert x \rvert$	(a) graph;	(b) range;	
3. $f(x) = \lvert \tan x \rvert$	(a) graph;	(b) range;	(c) period
4. $f(x) = [\![\sin x]\!]$	(a) graph;	(b) range;	(c) period
5. $f(x) = \lvert [\![\sin x]\!] \rvert$	(a) graph;	(b) range;	(c) period
6. $f(x) = [\![2 \sin x]\!]$	(a) graph;	(b) range;	(c) period
7. $f(x) = \frac{1}{2} [\![\lvert \sin x \rvert]\!]$	(a) graph;	(b) range;	
8. $f(x) = 2 [\![\lvert \sin x \rvert]\!]$	(a) graph;	(b) period	
9. $f(x) = 2 \lvert [\![\sin x]\!] \rvert$	(a) graph;	(b) period	
10. $-1 \leq y \leq \sin x$	(a) graph;	(b) range	
11. $y \geq \csc x$	(a) graph;	(b) range	
12. $0 \leq y \leq \cos x$	(a) graph;		
13. $y = \sin^2 x$	(a) graph;	(b) period;	(c) range

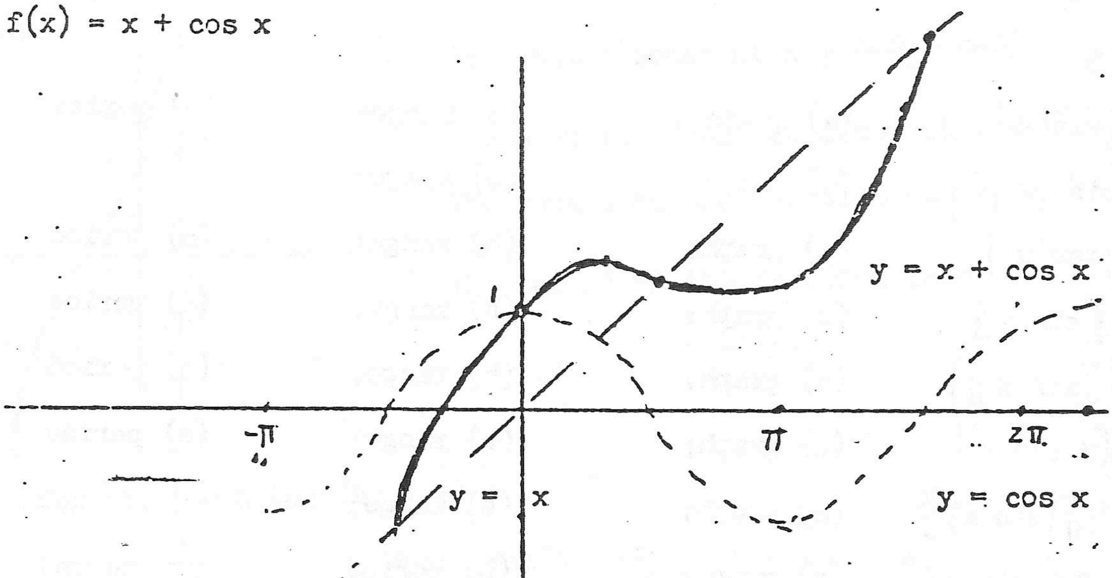
SECTION IV. GRAPHING BY ADDITION OF ORDINATES

Sometimes functions are defined in terms of the sum of two functions. For instance, $f(x) = \sin x + \cos x$. One could procede to graph by systematically plotting points. An alternate method is to sketch both graphs; i.e. $y = \sin x$ and $y = \cos x$, and then graphically add the ordinates. In the following examples an attempt is made to illustrate the method. Study the examples. Should things not be clear, have a chat with your instructor or someone else in the know!

EXAMPLE: $f(x) = \sin x + \cos x$



EXAMPLE: $f(x) = x + \cos x$



ASSIGNMENT 4 Graph as many of the following using the addition of ordinate method, as you need to to feel comfortable.

1. $f(x) = \frac{1}{2}x + \sin x$
2. $f(x) = \cos 2x + x$
3. $f(x) = 2 \sin x + 3$
4. $f(x) = x^2 + 1$
5. $f(x) = |x| + x$
6. $f(x) = |x| + \sin x$

SECTION V. EVALUATION

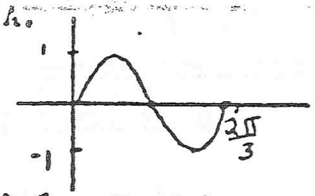
1. Review the L.A.P.
2. Take the Trial Run.
3. Take the Test.

GRAPHING ANSWERS

ASSIGNMENT 1.

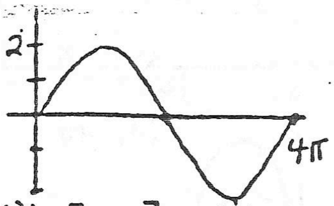
1. 1. (c); 2. (e); 3. (a); 4. (b); 5. (c); 6. (f); 7. (h); 8. (d).

2. A.



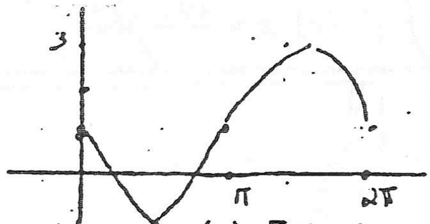
(a) [-1,1] (b) 1
(c) $\frac{2\pi}{3}$; (d) 0

B.



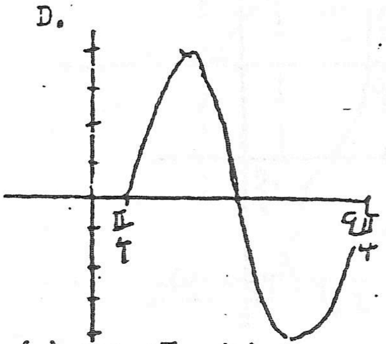
(a) [-2,2] (b) 2
(c) 4π; (d) 0

C.



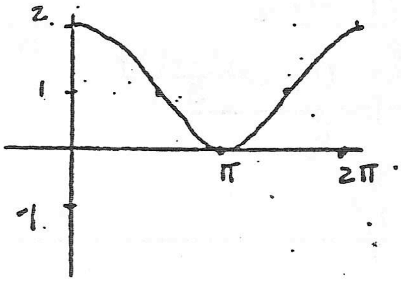
(a) [-1,3]
(b) 2; (c) 2π; (d) 0

D.



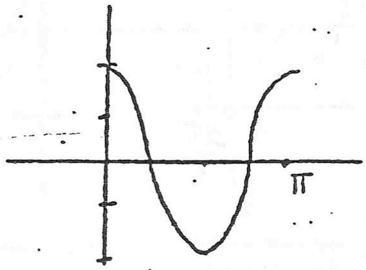
(a) [-4,4] (b) 4
(c) 2π; (d) $\frac{\pi}{4}$ right

E.



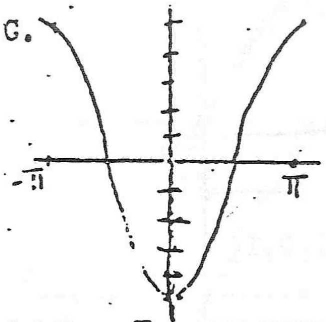
(a) [0,2] (b) 1
(c) 2π (d) 0

F.



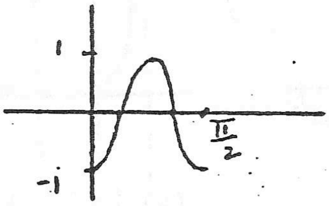
(a) [-2, 2] (b) 2
(c) π (d) 0

G.



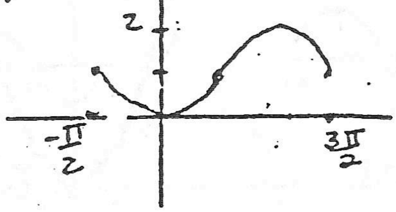
(a) [-5,5] (b) 5
(c) 2π (d) π, left

H.



(a) [-1,1] (b) 1
(c) $\frac{\pi}{2}$ (d) 0 relative to $y = -\cos x$;
(d) $\frac{\pi}{4}$ relative to $y = \cos x$

I.



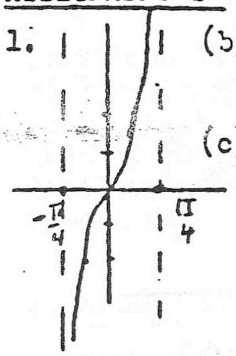
(a) [0, 2] (b) 1
(c) 2π;
(d) $\pi/2$ left rel. to $y = -\sin x$
$\pi/2$ right rel. to $y = \sin x$

3. $y = \sin(x - \frac{\pi}{4})$; 4. $y = 7 \cos x$; 5. $y = \sin(\frac{x}{2} - \frac{\pi}{2})$

6. $y = -\sin x + 3$

ASSIGNMENT 2

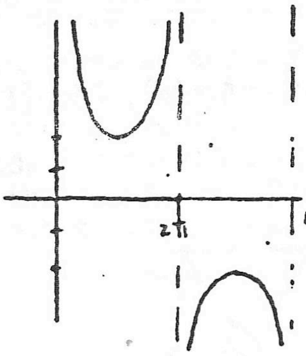
1.



(b) $x = \frac{k\pi}{2}$

(c) $x = \frac{(2k+1)\pi}{4}$

2.



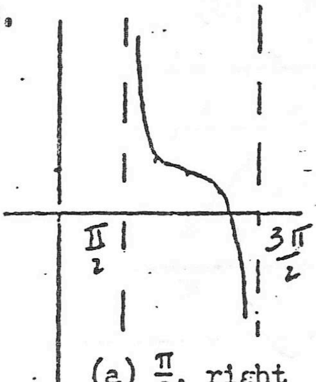
(a) $\{ y : |y| \geq 2 \}$

(b) $\{ x : x \neq 2k\pi \}$

(c) $x = 2k\pi$

(d) 4π

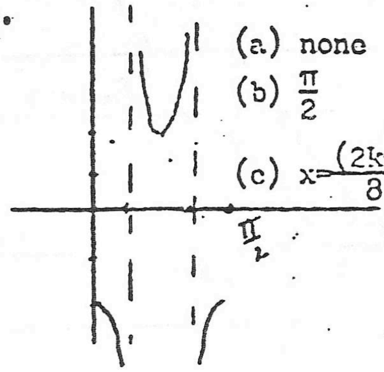
3.



(a) $\frac{\pi}{2}$, right

(b) π

4.

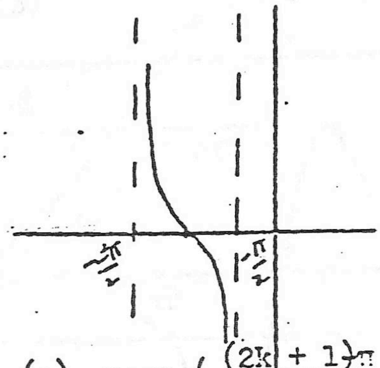


(a) none

(b) $\frac{\pi}{2}$

(c) $x = \frac{(2k+1)\pi}{8}$

5.

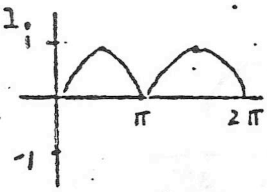


(a) $x : x \neq \frac{(2k+1)\pi}{2}$

(b) π

ASSIGNMENT 3

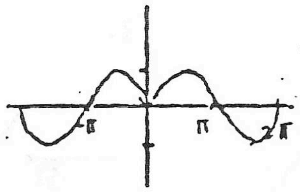
1.



(b) $[0,1]$

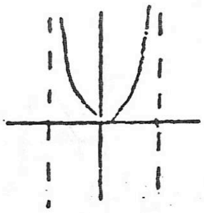
(c) π

2.



(b) $[-1,1]$

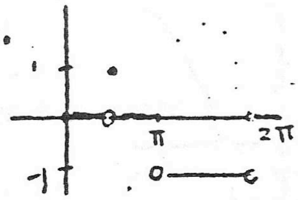
3.



(b) $[0, \infty)$

(c) π

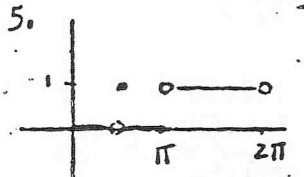
4.



(b) $\{-1,0,1\}$

(c) 2π

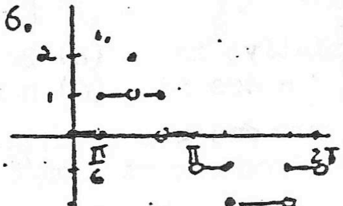
5.



(b) $\{0,1\}$

(c) 2π

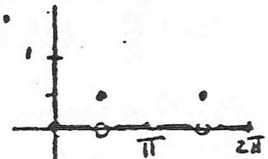
6.



(b) $\{-2,-1,0,1,2\}$

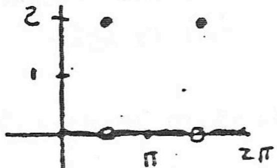
(c) 2π

7.



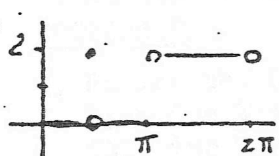
(b) $\{0,\frac{1}{2}\}$

8.



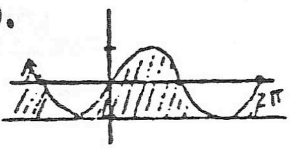
(b) π

9.



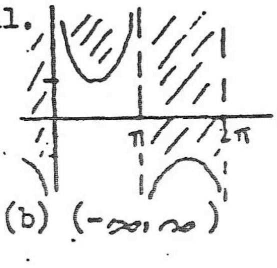
(b) 2π

10.



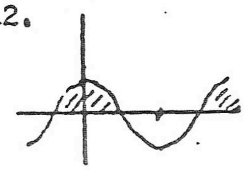
(b) $[-1,1]$

11.

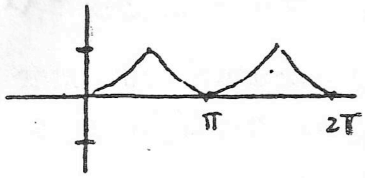


(b) $(-\infty, \infty)$

12.



13.

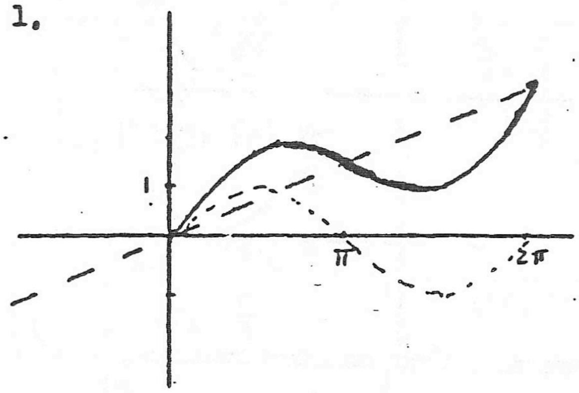


(b) π

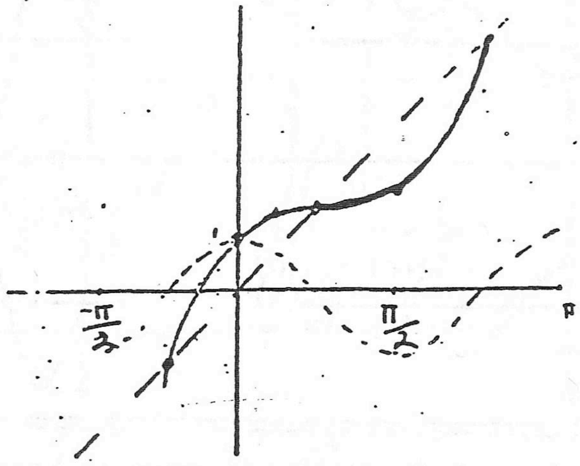
(c) [0, 1]

ASSIGNMENT 4

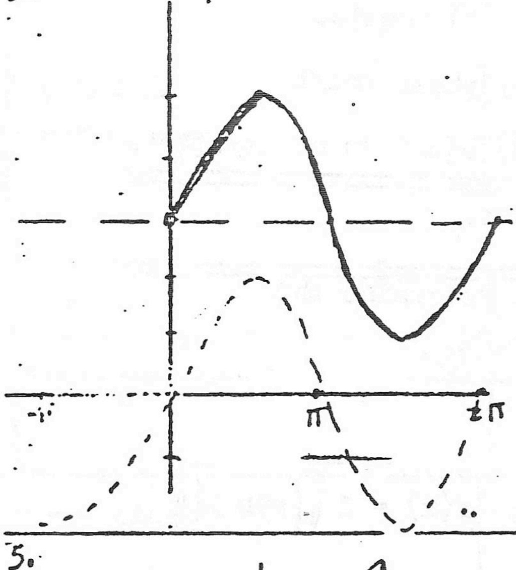
1.



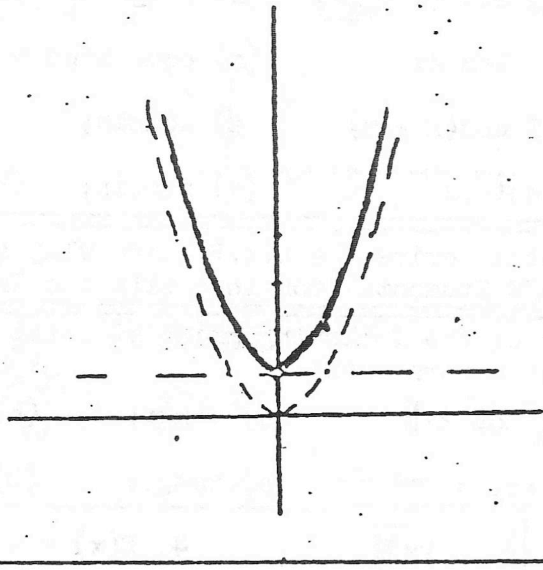
2.



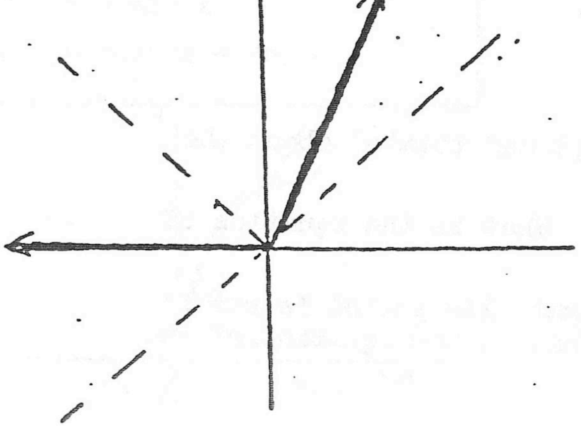
3.



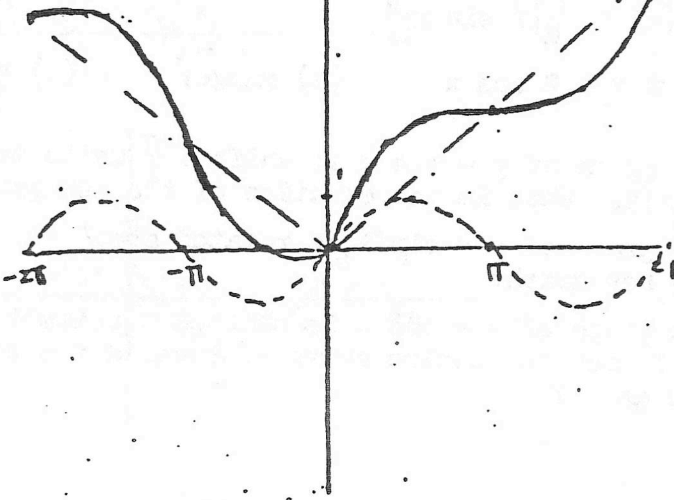
4.



5.



6.



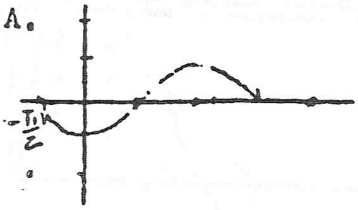
MATH ANALYSIS
TRIGONOMETRY

GRAPHS TRIAL RUN

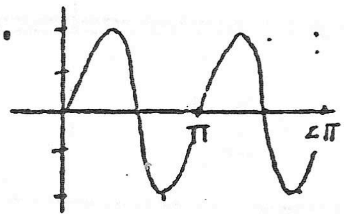
I. Match each of the following functions with their graphs:

1. $f(x) = \sin(x + \frac{\pi}{2})$
2. $f(x) = 2\sin 2x$
3. $f(x) = 2\sin \frac{1}{2}x$
4. $f(x) = \sin x + 1$
5. $f(x) = -\sin(x + \frac{\pi}{2})$
6. $f(x) = -2\sin 2x$

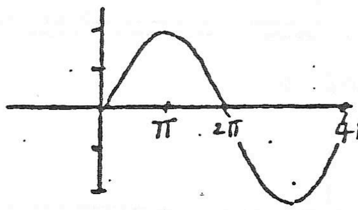
A.



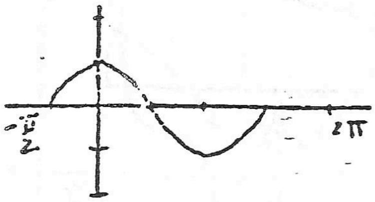
B.



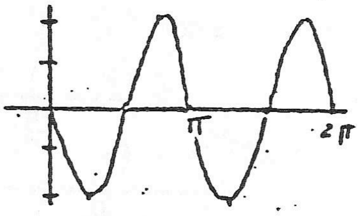
C.



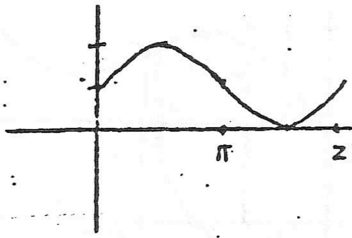
D.



E.



F.



II. For each of the following graph at least one period and determine the indicated information:

1. $f(x) = 3\sin(x + \frac{\pi}{2})$; (a) range; (b) amplitude; (c) period;
2. $f(x) = \frac{1}{2}\tan 2x$ (a) equations of asymptotes; (b) domain.
3. $f(x) = 2\csc(x - \pi)$ (a) domain; (b) range; (c) phase shift.
4. $f(x) = \cot 2x$ (a) domain; (b) period; (c) equation of asymptotes

TRIG Students: review the L.A.P. carefully and take the test.
MATH ANALYSIS Students: continue with the Trial Run.

III. For each of the following graph at least one period and determine the indicated information:

1. $f(x) = [\![\cos 2x]\!]$ (a) range; (b) period;
2. $f(x) = \cos^2 x$ (a) range; (b) period
3. $f(x) = |[\![\sin 2x]\!]|$
4. $f(x) = 2|[\![\sin x]\!]|$
5. $f(x) = 2[\![|\sin x|]\!]$
6. $f(x) = [\![|2\sin x|]\!]$.
7. $0 \le y < 2\cos x$ (a) range; (b) period

IV. The graph of $y = \sin x$ is shifted $\frac{\pi}{2}$ units to the left and rotated about the X-axis. What is the equation of the new graph?

2. The graph of $y = \sin x$ is rotated about the X-axis. What is the equation of the new graph?

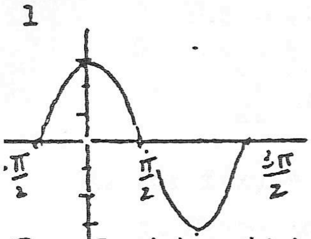
3. The graph of $y = \cos x$ is shifted π units to the right, its period is cut in half, and the entire curve is lowered two units. What is the equation of the new graph?

1. $f(x) = \sin^2 x + \cos^2 x$ 2. $f(x) = 2x + \sin x$ $f(x) = -\sin x + \cos x$

APHING TRIAL RUN ANSWERS

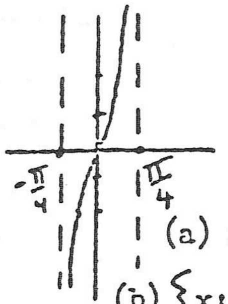
1. D; 2. B. 3. C; 4. F; 5. A; 6. E.

1



)[-3,3]; (b) 3; (c) 2π

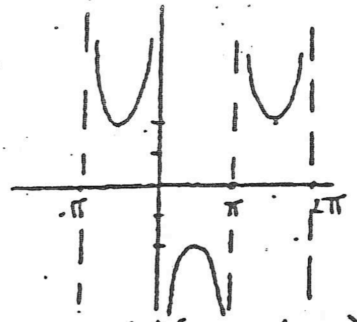
2.



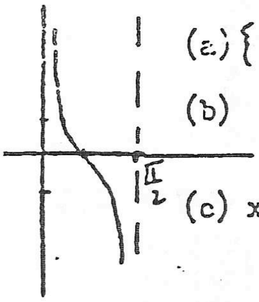
(a) $x = \frac{(2k+1)\pi}{4}$

(b) $\left\{x: x \neq \frac{(2k+1)\pi}{4}\right\}$

3.



(a) $\{x: x \neq k\pi\}$
(b) $\{y: |y| \geq 2\}$
(c) π right

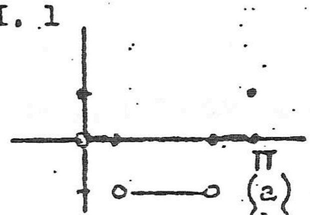


(a) $\{x | x \neq \frac{k\pi}{2}\}$

(b) $\frac{\pi}{2}$

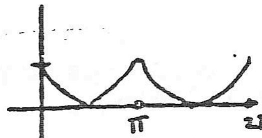
(c) $x = \frac{k\pi}{2}$

III. 1

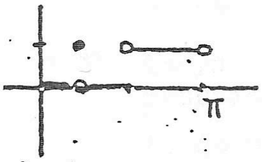


(a) -1,0,1
(b) π

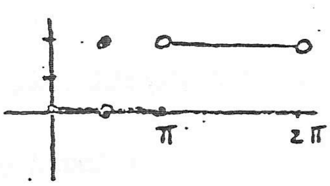
2.



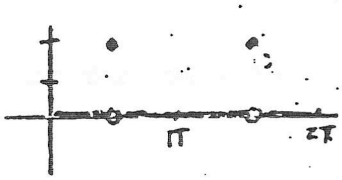
(a) [0,1]
(b) π

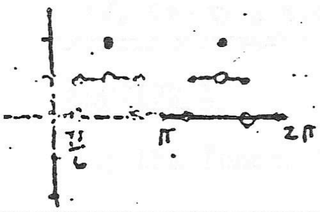


4

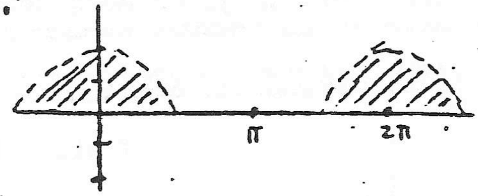


5.





7.



(a) 0, 2)

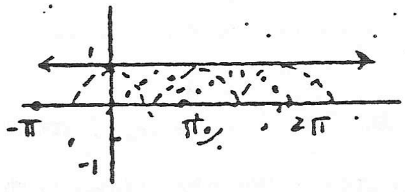
(b) 2π

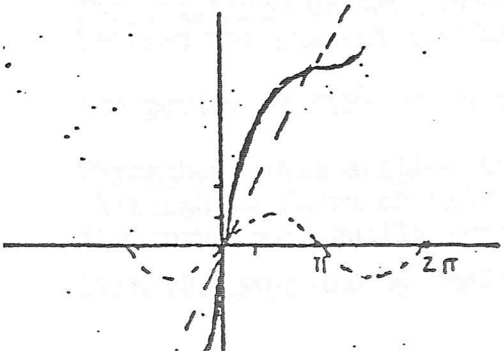
1 $y = -\sin(x + \frac{\pi}{2})$

2. $y = -\sin x$

3. $y = \cos(2x - 2\pi) - 2$

V. 1





3.