MATH ANALYSIS — INVERSE TRIG FUNCTIONS — Suggested Time: 2 weeks

*TRIGONOMETRY

Behavioral Objectives

I. Identify the domain and range of

A. y = sin x
B. y = Sin x
C. y = arc sin x
D. y = Arc sin x

II. Repeat I for the remaining five trig functions

III. Graph each relation discussed in I and II.

IV. Evaluate expressions involving the

A. Converses of the trig functions
B. Inverses of the trig functions

V. Solve equations involving the inverses of the trig functions

VI. Using the techniques of composition of functions

A. Evaluate expressions involving trig functions and their inverses
B. Solve equations involving trig functions and their inverses

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SECTION I — CONVERSES OF THE TRIG FUNCTIONS

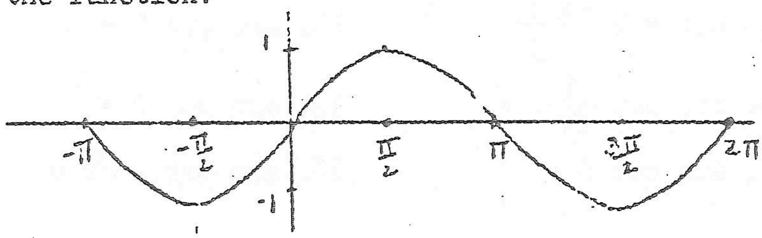
A trig function, like all relations has a converse. The converse of any function is formed simply by reversing the components of the ordered pairs of the function. For example, if the ordered pair $(\pi, 0)$ belongs to the sin function, then the ordered pair $(0, \pi)$ belongs to the converse of the sin function. Due to the periodic nature of the trig functions, none of them, in the true sense of the word, have inverses. Recall, a function has an inverse only under the condition that its converse is a function.

To speak of the converse of the trig functions we use the prefix: arc.

Note:

FUNCTION	CONVERSE	FUNCTION	CONVERSE
y = sin x	y = arc sin x	y = sec x	y = arc sec x
y = cos x	y = arc cos x	y = csc x	y = arc csc x
y = tan x	y = arc tan x	y = cot x	y = arc cot x

First consider the graph of y = sin x; the domain of the function; the range of the function:



Domain: $(-\infty, \infty)$

Range: $[^{-}1, 1]$

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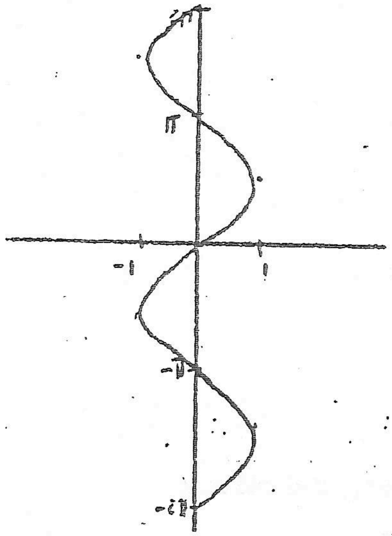
*TRIGONOMETRY STUDENTS: Do Sections III, IV, and V only.

Now, to graph y = arc sin x, merely reverse the components of the ordered pairs of y = sin x. This, of course, interchanges the domain and range.

GRAPH: y = arc sin x



Domain: $[-1, 1]$

Range: $(-\infty, \infty)$

ASSIGNMENT 1

Graph and determine the domain and range of each of the following relations:

1. y = arc cos x
2. y = arc tan x
3. y = arc sec x
4. y = arc csc x
5. y = arc cot x

SECTION II. EVALUATION OF EXPRESSIONS INVOLVING TRIG CONVERSES

When evaluating an expression such as arc cos $\frac{1}{2} = \theta$, we ask the question: For what values of θ is cos θ equal to $\frac{1}{2}$? When observing the unit circle we notice that cos $\theta = \frac{1}{2}$ for $\theta = \pi/3$ or $-\pi/3$. These two solutions are the primary solutions.

The complete solution is: $\theta = 2k\pi \pm \frac{\pi}{3}$.

ASSIGNMENT 2

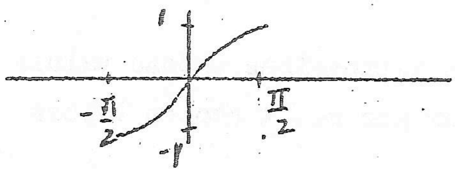
Evaluate each of the following: Give the complete solution.

1. arc sin 0 =
2. arc sin $\frac{\sqrt{3}}{2}$ =
3. arc sin $-\frac{\sqrt{3}}{2}$ =
4. arc sin 1 =
5. arc sin $\frac{\sqrt{2}}{2}$ =
6. arc tan $\frac{\sqrt{3}}{3}$ =
7. arc sin -1 =
8. arc csc 1 =
9. arc sec 1 =
10. arc sec 2 =
11. arc csc $-\frac{\sqrt{2}}{3}$
12. arc sin 2 =
13. arc tan -1 =
14. arc cot $\frac{1}{\sqrt{3}}$ =
15. arc cos 0 =
16. arc tan 0 =
17. arc cot 0 =
18. arc sec $-\sqrt{2}$ =

In order to actually consider an inverse of a trig function it is necessary to restrict the domain of each trig function. The restriction is somewhat artitrary. We shall use the customary restrictions.

For the sin function: The restricted sin function is noted by using a capital S on the word Sin. For the function y = Sin x, the domain is $-\pi/2, \pi/2$

The Graph of y = Sin x

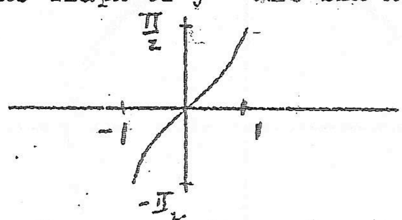


Domain: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Range: $[-1, 1]$

The inverse of y = Sin x is y = Arc sin x. Notice the use of capital A for the word Arc sin x.

The Graph of y = Arc sin x



Domain: $[-1, 1]$

Range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Another commonly used notation for y = Arc sin x is $y = \text{Sin}^{-1}x$. $\text{Sin}^{-1}x$ is read Sin invers x. The negative 1 exponent is used here in the functional sense. Recall $f(x)$ and $f^{-1}(x)$.

BEWARE: $\text{Sin}^{-1} x \neq \frac{1}{\text{Sin } x}$

Each of the standard trig functions must have its domain restricted before corresponding inverse trig functions can be considered. The following are the accepted domain restrictions. It is important that you become very familiar with each function and how it is restricted.

Function	Domain
y = Sin x	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
y = Cos x	$[0, \pi]$
y = Tan x	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
y = Sec x	$\left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$
y = Csc x	$\left[-\frac{\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$
y = Cot x	$(0, \pi)$

ASSIGNMENT 3

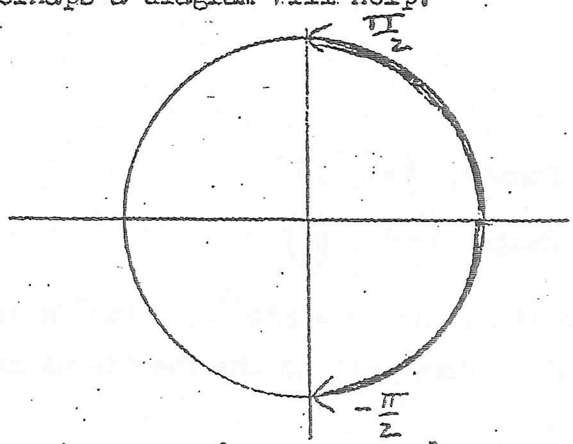
1. Graph each of the Trig functions over its restricted domain.
2. Determine the range of each of the functions graphed in #1.
3. Graph the inverse of each Trig function.
4. Determine the domain and range of each of the functions graphed in #3.

SECTION IV. EVALUATION OF EXPRESSIONS INVOLVING TRIG INVERSES

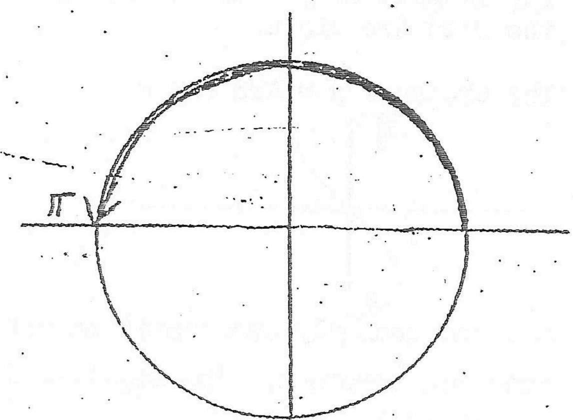
When evaluating expressions involving trig inverses, procede as in Section II. Here, however, the solution is unique.

When evaluating Arc sin x, Arc tan x, or Arc csc x expressions choose values between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$. For Arc cos x, Arc cot x, or Arc sec x choose values between 0 and π.

Perhaps a diagram will help.



For: $y = \text{Sin}^{-1}x$; $y = \text{Tan}^{-1}x$; $y = \text{Csc}^{-1}x$



For: $y = \text{Cos}^{-1}x$; $y = \text{Cot}^{-1}x$; $y = \text{Sec}^{-1}x$

Examples:

1. Arc cos $\frac{1}{2} = \frac{\pi}{3}$; 2. $\text{Sin}^{-1} -\frac{1}{2} = -\frac{\pi}{6}$; 3. Arc tan $-1 = -\frac{\pi}{4}$

ASSIGNMENT 4

Evaluate each of the following:

1. Arc sin 1 =	2. Arc cos -1 =	3. Arc sin -1 =
4. Csc^{-1} 2 =	5. Arc cos 0 =	6. Arc sec $\frac{2\sqrt{3}}{3}$ =
7. Cot^{-1} 1 =	8. $\text{Tan}^{-1}\sqrt{3}$ =	9. Arc sin $\frac{\sqrt{3}}{2}$ =
10. $\text{Cos}^{-1} -\frac{1}{2}$ =	11. $\text{Csc}^{-1} -\sqrt{2}$ =	12. Arc cot 0 =
13. Tan^{-1} 1 =	14. $\text{Tan}^{-1} -\frac{\sqrt{3}}{3}$ =	15. Sec^{-1} -1 =
16. Arc sin 3 =	17. Arc cos -1 =	18. Arc csc -2 =

SECTION V. COMPOSITION OF FUNCTIONS

Consider an expression like: cot(Arc tan -1). To evaluate such an expression one wishes to find the cot of the number whose tangent is -1. That number must be between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ because the values of Arc tan are in that range.

So we have: $\cot(\text{Arc tan } -1) = \cot\left(-\frac{\pi}{4}\right) = -1$

Consider each of the following examples and then procede to the assignment.

Example 1: Evaluate $\text{Sin}^{-1}(\cos(\text{Arc csc } 2))$.

Solution: $\text{Sin}^{-1}(\cos(\text{Arc csc } 2)) = \text{Sin}^{-1}\left(\cos\frac{\pi}{6}\right)$

$= \text{Sin}^{-1}\frac{\sqrt{3}}{2}$

$= \underline{\underline{\frac{\pi}{3}}}$

Example 2: Evaluate: $\text{Tan}^{-1}\left(\tan\frac{3\pi}{4}\right)$

Solution: $\text{Tan}^{-1}\left(\tan\frac{3\pi}{4}\right) = \text{Tan}^{-1}\ -1$

$= \underline{\underline{-\frac{\pi}{4}}}$

Example 3: Evaluate: $\cos(\text{Arc tan } \frac{3}{4})$

Solution: Let $\text{Arc tan } \frac{3}{4} = \theta$ for $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

Then: $\text{Tan}(\text{Arc tan } \frac{3}{4}) = \text{Tan } \theta$ [Take Tan of both sides]

$\frac{3}{4} = \text{Tan } \theta$ [Tan and Arc Tan are inverses so... zap.]

Our problem has become one of determining cos θ when we know Tan θ = 3/4.

Recall: $\tan^2\theta + 1 = \sec^2\theta$

Hence: $9/16 + 1 = \sec^2\theta$

$25/16 = \sec^2\theta$

$5/4 = \sec\theta$ [Reject -5/4 because tan θ is positive and therefore we are in the first quadrant.]

Thus: $\cos\theta = \underline{\underline{4/5}}$

Example 4: Evaluate: tan Arc sin cos Arc csc 2

Solution: $\tan \text{Arc} \sin \cos \text{Arc} \csc 2 = \tan \text{Arc} \sin \cos \frac{\pi}{6}$

$= \tan \text{Arc} \sin \frac{\sqrt{3}}{2}$

$= \tan \frac{\pi}{3}$

$= \sqrt{3}$

Example 5: Evaluate $\cos \text{Arc} \sin \frac{8}{9}$

Solution: Since we don't know just where the value of the sin is $\frac{8}{9}$,

let $\theta = \text{Arc} \sin \frac{8}{9}$ Then: $\sin \theta = \sin(\text{Arc} \sin \frac{8}{9})$

$\sin \theta = \frac{8}{9}$

Our problem has become one of finding the cos of θ when we know $\sin \theta = \frac{8}{9}$.

$\sin^2\theta + \cos^2\theta = 1$; hence $\frac{64}{81} + \cos^2\theta = 1$

$\cos^2\theta = \frac{17}{81}$

$\cos \theta = \frac{\sqrt{17}}{9}$

Hence: $\cos \text{Arc} \sin \frac{8}{9} = \frac{\sqrt{17}}{9}$

ASSIGNMENT 5

Evaluate each of the following:

1. $\sin (\text{Cos}^{-1} \frac{\sqrt{3}}{2})$
2. $\sin(\text{Arc} \cos -1)$
3. $\cos(\text{Sin}^{-1} 0)$
4. $\cot (\text{Arc} \tan \frac{\sqrt{2}}{2})$
5. $\csc(\text{Arc} \cos -1)$
6. $\sin(5(\text{Arc} \sin 1))$
7. $\sec \text{Arc} \sin \frac{1}{2}$
8. $\text{Arc} \cos \sin \frac{3\pi}{4}$
9. $\sin \text{Arc} \sin -\frac{1}{2}$
10. $\cot \text{Arc} \tan \sqrt{3}$

12. $2 \sin 2 \text{Sin}^{-1} \frac{1}{2}$
13. $\frac{1}{2} \cos \frac{1}{2} \text{Cos}^{-1} 0$

14. Arc sin tan Arc cos $\frac{-\sqrt{2}}{2}$

15. cos Arc sin $\frac{8}{17}$

16. cos Arc tan $\frac{-5}{12}$

17. sin Arc cos $\frac{1}{10}$

18. tan Arc sin $- \frac{1}{2}$

19. sin Arc tan $\frac{3}{4}$

20. cot Arc tan 1

21. sin Arc tan -1

22. 2 sin 2 Arc sin $\frac{1}{2}$

23. Arc sin cos Arc csc 2

24. cos 3 Arc sin $\frac{1}{2}$

25. Arc cos tam Arc sin $\frac{-\sqrt{2}}{2}$

SECTION VI. INVERSE TRIGONOMETRIC EQUATIONS

This final section deals with equations involving trig inverses. It will be to apply good algebra and correct trig relationships in order to achieve solutions for the given equations. Study the examples below and procede to the assignment.

<u>Example 1</u>: Arc tan x = Arc tan $\frac{1}{4}$ - Arc tan $\frac{1}{2}$

<u>Back room stuff</u>

Let Arc tan $\frac{1}{4} = \theta$

Then Take the tan of both sides so:

$\frac{1}{4} = \tan\theta$

Let Arc tan $\frac{1}{2} = \emptyset$

Then $\frac{1}{2} = \tan\emptyset$

(after the back room)

Arc tan x = $\theta - \emptyset$

$\tan(\text{Arc tan } x) = \tan(\theta - \emptyset)$

$$x = \frac{\tan\theta - \tan\emptyset}{1 + \tan\theta\tan\emptyset}$$

$$x = \frac{\frac{1}{4} - \frac{1}{2}}{1 + \frac{1}{4}\cdot\frac{1}{2}}$$

$$x = \frac{-\frac{1}{4}}{\frac{9}{8}}$$

$$x = -\frac{2}{9}$$

<u>Example 2</u>: Arc sin $\frac{x}{2}$ = Arc cos $\frac{\sqrt{3}}{2}$ . . . Solve for x:

Solution: Arc sin $\frac{x}{2} = \frac{\pi}{6}$ Comments: [Arc cos $\frac{\sqrt{3}}{2} = \frac{\pi}{6}$]

$\frac{x}{2} = \sin\frac{\pi}{6}$ [Take the sin of both sides]

$\frac{x}{2} = \frac{1}{2}$ [$\sin\frac{\pi}{6} = \frac{1}{2}$]

$x = 1$ [algebra]

Example 3: Solve for x:

$$\text{Arc sin } x = \text{Arc cos } 2x$$

$\theta = \text{Arc cos } 2x$ [Let Arc sin x = θ]

$\cos\theta = 2x$ [Take the cos of both sides]

$\sin\theta = x$ [Since Arc sin x = θ; take sin of both sides.]

$x^2 + (2x)^2 = 1$ [$\sin^2\theta + \cos^2\theta = 1$]

$x^2 + 4x^2 = 1$

$5x^2 = 1$

$x = \frac{1}{\sqrt{5}}$

ASSIGNMENT 6

Solve each of the following for x:

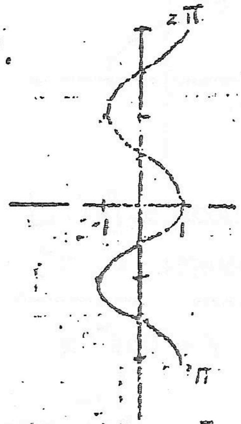
1. $\text{Arc cos } 2x = \text{Arc sin } \frac{\sqrt{3}}{2}$
2. $\text{Arc cos } \frac{x}{2} = \text{Arc sin } \frac{\sqrt{3}}{2}$
3. $\text{Arc sin } x = \text{Arc cos } \frac{x}{2}$
4. $2 \text{ Arc cos } x + \text{Arc sin } x = \text{Arc cos } -\frac{1}{2}$
5. $\text{Arc sin } x = \text{Arc csc } 2$
6. $\text{Arc tan } x = \text{Arc tan } \frac{3}{4} + \text{Arc tan } \frac{5}{7}$
7. $\cos(\frac{1}{2} \text{ Arc sin } \frac{5}{12}) = x$
8. $\cos(2 \text{ Arc sin } -\frac{5}{12}) = x$
9. $\cos(\text{Arc tan } \frac{9}{5}) = x$
10. $\text{Cos}^{-1} x = \text{Sin}^{-1} \frac{x}{2}$
11. $2 \text{ Arc cos } 2x = \frac{\pi}{2}$
12. $\sin(2 \text{ Arc cos } \frac{5}{7}) = x$
13. $\text{Cos}^{-1} 3x = \text{Sin}^{-1} x$
14. $\text{Arc tan } x + \text{Arc tan } 2x = \text{Arc tan } \frac{4}{3}$

SECTION VII. EVALUATION

1. Review the Behavioral Objectives
2. Take the Trial Run.
3. Take the test.

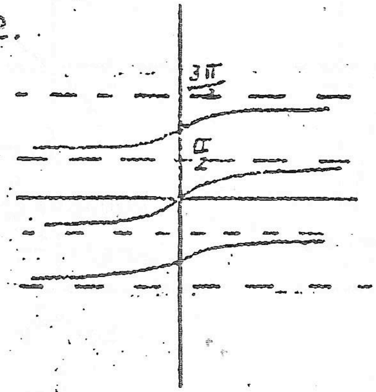
ANSWERS Courtesy of the Answer Grape

1.



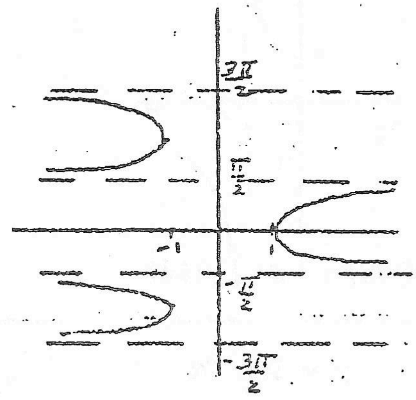
Domain: $[-1, 1]$
Range: $(-\infty, \infty)$

2.



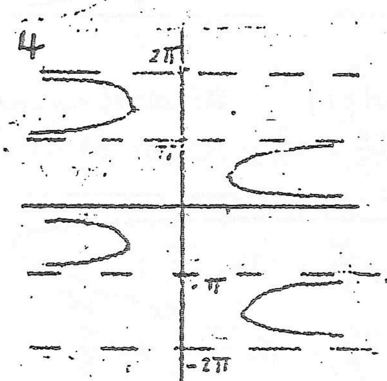
Domain: $(-\infty, \infty)$
Range: $\left\{y: y \neq \frac{(2k+1)\pi}{2}\right\}$

3.



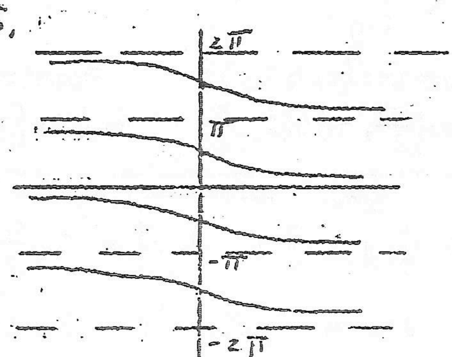
Domain: $\{x: |x| \geq 1\}$
Range: $\left\{y: y \neq \frac{(2k+1)\pi}{2}\right\}$

4



Domain: $\{x: |x| \geq 1\}$
Range: $\{y: y \neq k\pi\}$

5.



Doamin: $(-\infty, \infty)$
Range: $\{y: y \neq k\pi\}$

ASSIGNMENT 2

1. $k\pi$; 2. $\frac{(4k+1)\pi}{2} \pm \frac{\pi}{6}$; 3. $\frac{(4k+3)\pi}{2} \pm \frac{\pi}{6}$; 4. $\frac{(4k+1)\pi}{2}$;

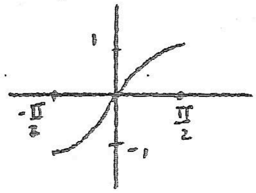
5. $\frac{(4k+1)\pi}{2} \pm \frac{\pi}{4}$; 6. $\frac{\pi}{6} + k\pi$; 7. $\frac{(4k+3)\pi}{2}$; 8. $\frac{(4k+1)\pi}{2}$; 9. $2k\pi$;

10. $2k\pi \pm \frac{\pi}{3}$; 11. No solution; 12. No solution; 13. $\frac{3\pi}{4} \pm k\pi$;

14. $\frac{\pi}{3} + k\pi$; 15. $\frac{(2k+1)\pi}{2}$; 16. $k\pi$; 17. $\frac{(2k+1)\pi}{2}$; 18. $(2k+1)\pi \pm \frac{\pi}{4}$.

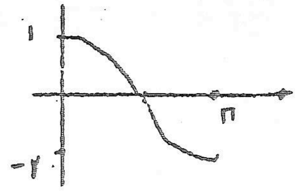
ASSIGNMENT 3

1. y = Sin x



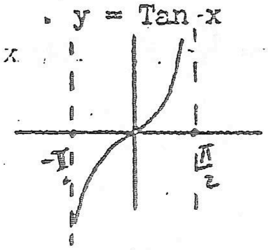
Range: $[-1, 1]$

. y = Cos x



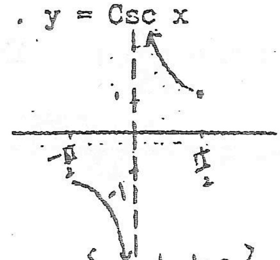
Range: $[-1, 1]$

. y = Tan x



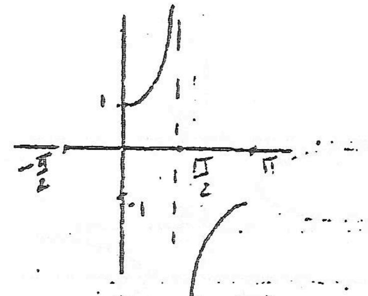
Range: $(-\infty, \infty)$

. y = Csc x



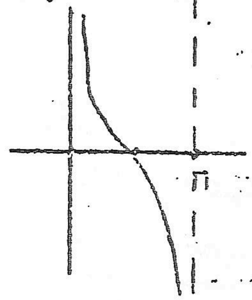
Range: $\{y: |y| \geq 1\}$

y = Sec x



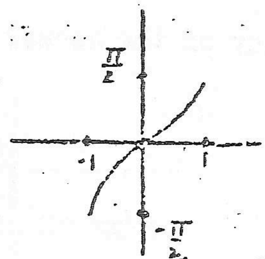
Range: $\{y: |y| \geq 1\}$

y = Cot x



Range: $(-\infty, \infty)$

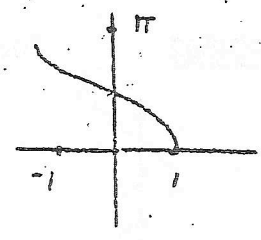
3. $y = \text{Sin}^{-1}x$



Domain: $[-1, 1]$

Range: $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

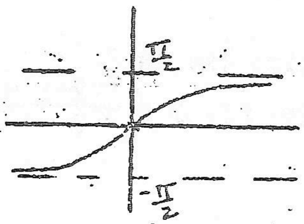
$y = \text{Cos}^{-1}x$



Domain: $[-1,1]$

Range: $[0, \pi]$

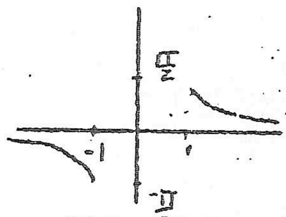
$y = \text{Tan}^{-1}x$



Domain: $(-\infty, \infty)$

Range: $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

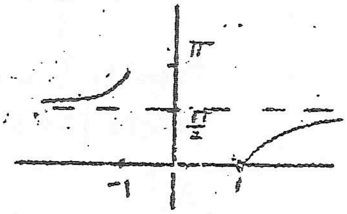
$y = \text{Csc}^{-1}x$



Domain: $\{x: |x| \geq 1\}$

Range: $\left[\frac{-\pi}{2}, 0\right) \cup \left(0, \frac{\pi}{2}\right]$

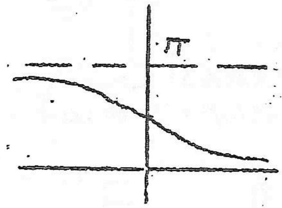
$y = \text{Sec}^{-1}x$



Domain: $\{x: |x| \geq 1\}$

Range: $\left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$

$y = \text{Cot}^{-1}x$



Domain: $(-\infty, \infty)$

Range: $(0, \pi)$

ASSIGNMENT 4 5.+6.

1. $\frac{\pi}{2}$; 2. π; 3. $\frac{-\pi}{2}$; 4. $\frac{\pi}{6}$; 7. $\frac{\pi}{4}$; 8. $\frac{\pi}{3}$; 9. $\frac{-\pi}{3}$; 10. $\frac{2\pi}{3}$; 11. $\frac{-\pi}{4}$;

12. $\frac{\pi}{2}$; 13. $\frac{\pi}{4}$; 14. $\frac{-\pi}{6}$; 15. π; 16. --; 17. π; 18. $\frac{-\pi}{6}$. 5. $\frac{\pi}{2}$; 6. $\frac{\pi}{3}$

ASSIGNMENT 5

1. $\frac{1}{2}$; 2. 0; 3. 1; 4. $\sqrt{2}$; 5. --; 6. 1; 7. $\frac{2}{\sqrt{3}}$; 8. $\frac{\pi}{4}$; 9. $-\frac{1}{2}$;

10. $\frac{1}{\sqrt{3}}$; 12. $\sqrt{3}$; 13. $\frac{\sqrt{2}}{4}$; 14. $\frac{-\pi}{2}$; 15. $\frac{15}{17}$; 16. $\frac{12}{13}$; 17. $\frac{3\sqrt{11}}{10}$

18. $\frac{-1}{\sqrt{3}}$; 19. $\frac{3}{5}$; 20. 1; 21. $\frac{-\sqrt{2}}{2}$; 22. $\sqrt{3}$; 23. $\frac{\pi}{3}$; 24. 0; 25. π.

ASSIGNMENT 6 The Answer Grape is sorry about these.

1. $x = \frac{1}{4}$; 2. $x = 1$; 3. $x = \frac{2}{\sqrt{5}}$; 4. $x = \frac{3}{2}$; 5. $x = \frac{1}{2}$;

6. $x = \frac{41}{13}$; 7. $x = \sqrt{\frac{12 + \sqrt{119}}{24}}$; 8. $x = \frac{47}{72}$; 9. $x = \frac{5}{\sqrt{106}}$;

10. $x = \frac{2}{\sqrt{5}}$; 11. $x = \frac{\sqrt{2}}{4}$; 12. $x = \frac{20\sqrt{6}}{49}$; 13. $x = \frac{1}{\sqrt{10}}$;

14. $x = \frac{-9 + \sqrt{209}}{16}$

I. (a) Graph; (b) State Domain; (c) State Range

1. $y = \text{Cos}^{-1}x$
2. $y = \text{Arc tan } x$
3. $y = \text{Sin } x$
4. $y = \text{Csc } x$
*5. $y = \text{arc sin } x$
*6. $y = \text{arc tan } x$

(The above are just sample graphs. You should be able to graph all functions and relations discussed in the L.A.P.)

II. Evaluate

1. Arc cos $\frac{-\sqrt{3}}{2}$
2. Arc tan -1
3. $\text{Sin}^{-1} \frac{1}{2}$
4. $\text{Tan}^{-1} 0$
5. Sin $\frac{\pi}{4}$
6. Sin $\frac{3\pi}{4}$
7. Cos $\frac{3\pi}{4}$
8. Cos $\frac{-\pi}{4}$
9. Arc tan $-\sqrt{3}$

*10. arc sin -1

*11. arc sec $\sqrt{2}$

*12. arc tan 0

*13. arc sin $\frac{\sqrt{2}}{2}$

*14. arc csc 1

*15. arc tan 1

III. Evaluate

1. sin Arc csc -1
2. tan Arc cot $-\sqrt{3}$
3. ~~sin~~ Arc cos $\frac{\sqrt{2}}{2}$
4. tan Arc sin 1
5. sec Arc cos $\frac{1}{2}$
6. $\text{Sin}^{-1} \cos \text{Csc}^{-1} 2$
7. $\cos(2 \text{ Arc sin } \frac{1}{2})$
8. $\sin(\frac{1}{2} \text{Cos}^{-1} \frac{1}{2})$
9. $\text{Sin}^{-1} \tan \text{Cos}^{-1} \frac{-\sqrt{2}}{2}$
10. cot Arc tan 1
11. sin Arc sin $\frac{1}{2}$
12. Arc cos $(\cos \frac{-\pi}{4})$

*IV. Solve each of the following for x:

1. $\text{Arc sin } \frac{\sqrt{2}}{2} - \text{Arc sin } \frac{1}{2} = x$
2. $\text{Cos}^{-1} 2x = \text{Sin}^{-1} x$
3. $\text{Arc cos}(2x^2 - 1) = 2 \text{ Arc cos } \frac{1}{2}$
4. $\text{Cos}^{-1} 2x - \text{Cos}^{-1} x = \frac{\pi}{3}$
5. $\text{Arc tan } x + \text{Arc tan } (1 - x) = \text{Arc tan } \frac{4}{3}$
6. $\text{Cos}^{-1} x + \text{Tan}^{-1} x = \frac{\pi}{2}$
7. $\text{Arc sin } \frac{1}{\sqrt{5}} + \text{Arc sin } \frac{2}{\sqrt{5}} = x$
8. $\sin(\text{Arc sin } \frac{12}{13} + \text{Arc sin } \frac{4}{5}) = x$
9. $\sin(\frac{1}{2} \text{Arc cos } \frac{4}{5}) = x$
10. $2 \text{ Arc cos } x + \text{Arc sin } 2x = \pi$
11. $2 \text{ Arc cos } x + \text{Arc sin } x = \frac{\pi}{2}$

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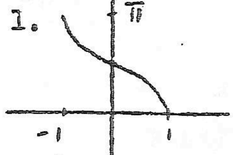
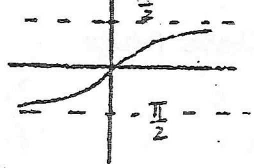
* Trig students omit these problems.

1.



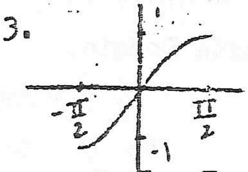
D: $[-1,1]$
R: $[0, \pi]$

2.



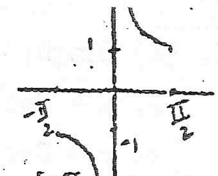
D: $(-\infty, \infty)$
R: $(\frac{-\pi}{2}, \frac{\pi}{2})$

3.



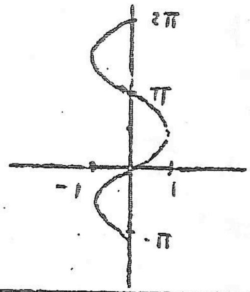
D: $[\frac{-\pi}{2}, \frac{\pi}{2}]$
R: $[-1, 1]$

4.



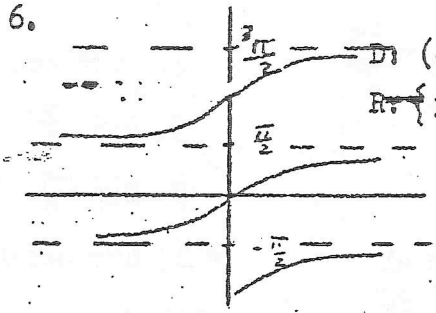
D: $[\frac{-\pi}{2}, 0) \cup (0, \frac{\pi}{2}]$
R: $\{y: |y| \geq 1\}$

5.



D: $[-1,1]$
R: $(-\infty, \infty)$

6.



D: $(-\infty, \infty)$
R: $\{y: y \neq \frac{(2k+1)\pi}{2}\}$

II. 1. $\frac{5\pi}{6}$; 2. $\frac{-\pi}{4}$; 3. $\frac{\pi}{6}$; 4. 0; 5. $\frac{\sqrt{2}}{2}$; 6. undefined;

7. $\frac{-\sqrt{2}}{2}$; 8. Undefined; 9. $\frac{-\pi}{3}$; 10. $\frac{(4k+3)\pi}{2}$; 11. $2k\pi \pm \frac{\pi}{4}$;

12. $k\pi$; 13. $\frac{(4k+1)\pi}{2} \pm \frac{\pi}{4}$; 14. $\frac{(4k+1)\pi}{2}$; 15. $k\pi + \frac{\pi}{4}$

III. 1. -1; 2. $-\frac{\sqrt{3}}{3}$; 3. $\frac{\sqrt{2}}{2}$; 4. undefined; 5. 2; 6. $\frac{\pi}{3}$; 7. $\frac{1}{2}$

8. $\frac{1}{2}$; 9. $\frac{-\pi}{2}$; 10. 1; 11. $\frac{1}{2}$; 12. $\frac{\pi}{4}$

IV. 1. $\frac{\pi}{12}$; 2. $\frac{1}{\sqrt{5}}$; 3. $\pm\frac{1}{2}$; 4. $-\frac{1}{2}$; 5. $\frac{1}{2}$; 6. 0;

7. $\frac{\pi}{2}$; 8. $\frac{56}{65}$; 9. $\frac{1}{\sqrt{10}}$; 10. 0; 11. 1.