Math Analysis 3 — **LINES**

Analytic Geometry — Learning Activities Package

BEHAVIORAL OBJECTIVES

I. Given the coordinates of two points determine

A. The equation of the line containing the two points
B. The length of the segment determined by the two points
C. The coordinates of the midpoint of the determined segment

II. Given the slope of a line and a point on the line determine

A. The equation of the line
B. The angle of inclination of the line
C. The X-intercept of the line
D. The Y-intercept of the line

III. Given the equation of a line determine

A. The slope of the line
B. The X-intercept of the line
C. The Y-intercept of the line
D. The angle of inclination of the line
E. The graph of the line
F. The equation of a line through a given point
1. Parallel to the given line
2. Perpendicular to the given line

IV. Find the distance

A. Between two points
B. From a point to a line
C. Between two parallel lines

V. Find the measures of the angles between two intersecting lines

* VI. Use coordinate geometry to prove selected plane geometry theorems. (See Section V. of the L.A.P.)

SECTION I — A REVIEW OF THE BASICS OF COORDINATE GEOMETRY

Most of the concepts of this section should not be new to the reader. We review them because they are important for subsequent work, and we really don't expect total recall of all previous math experience!

Suppose we are given two points A: (-2,3) and B: (5,9).

1. We can determine the length of segment $\overline{AB}$ by using the distance formula.

$$AB = \sqrt{(-2-5)^2 + (3-9)^2} = \sqrt{49+36} = \sqrt{85}$$

2. The midpoint of $\overline{AB}$ is $(\frac{3}{2}, 6)$. The midpoint coordinates are the averages of the endpoint coordinates.

3. The slope of $\overleftrightarrow{AB}$ is the ratio of the change in the y coordinates to the change in the x coordinates.

Sometimes the slope is written as $\frac{\Delta y}{\Delta x}$, or $\frac{y_1 - y_2}{x_1 - x_2}$.

For our example, the slope of line $\overleftrightarrow{AB}$ is $\frac{6}{7}$.

4. To determine the equation of AB we choose an arbritrary point (x,y) on the line. We use the fact that the slope of a line is constant. Thus:

$$\frac{y-9}{x-5} = \frac{6}{7} \qquad (1)$$

The above equation can be simplified to:

$$7y - 63 = 6x - 30$$

$$7y = 6x + 33$$

$$y = \frac{6}{7}x + \frac{33}{7} \qquad (2)$$

(1) is a perfectly good line equation. It is the immediate result when one determines a line equation when given two points on the line.

(2) is called the slope-intercept form of a line equation. The coefficient of x is the slope of the line and the final number is the Y-intercept of the line.

5. To find the X-intercept of line AB, use equation (2) and let $y = 0$. The X-intercept is $-\frac{33}{6}$.

6. Some notes: (a) Parallel lines have equal slopes and different Y-intercepts.
(b) If two lines are perpendicular, the product of their slopes is negative 1, or one migth say, that their slopes are negative reciprocals of each other.

Since this section is primarily a review, perhaps the best thing to do is become active in solving some problems. You may require some help from your friends or a teacher. Get that help if you need it. Make sure that you are an <u>expert</u> in the work that follows before you go on to section II.

EXERCISE I

1. Given A: (2,3) and B: (8,-7) determine:
   (a) The length of $\overline{AB}$
   (b) The midpoint of $\overline{AB}$
   (c) The slope of $\overleftrightarrow{AB}$
   (d) The equation of $\overleftrightarrow{AB}$
   (e) The X-intercept of $\overleftrightarrow{AB}$
   (f) The Y-intercept of $\overleftrightarrow{AB}$
   (g) The equation of the line which contains (4,12) and is parallel to $\overleftrightarrow{AB}$.
   (h) The equation of the line which contains (4,12) and is perpendicular to $\overleftrightarrow{AB}$.

2. The equation of $\overleftrightarrow{BC}$ is $y = 7x + 8$
   (a) The slope of $\overleftrightarrow{BC}$ is ___
   (b) The Y-intercept of $\overleftrightarrow{BC}$ is _____
   (c) The X-intercept of $\overleftrightarrow{BC}$ is ____
   (d) If $\overleftrightarrow{DE}$ is parallel to $\overleftrightarrow{BC}$ and contains the point (0,0), the equation of $\overleftrightarrow{DE}$ is?
   (e) If $\overleftrightarrow{FG}$ is perpendicular to $\overleftrightarrow{BC}$ and contains (1,2), the equation of $\overleftrightarrow{FG}$ is?

3. Write the equation of each line as identified below:
   (a) Contains the points (3,8) and (-2,-1).
   (b) Slope 2 and contains the point (1,4).
   (c) Slope 0 and contains the point (2,5).
   (d) Slope $-\frac{1}{2}$ and Y-intercept 4.
   (e) Slope $-\frac{1}{2}$ and X-intercept 4.
   (f) X-intercept 4 and Y-intercept 2.
   (g) Parallel to the Y-axis and contains the point (8,-2).
   (h) Y-intercept 2 and contains the point (2,9).

4. Find the equation of the perpendicular bisector of the segment with endpoints (2,8) and (-6,10).

5. Find the equation of the line through (3,-2) and
   (a) parallel to the line $x + 5y - 3 = 0$.
   (b) perpendicular to the line $x + 5y - 3 = 0$.

6. Show that the triangle formed by joining the points (2,1), (5,2) and (4,5) is isosceles. What is the area of the triangle?

7. Show that the triangle formed by joining (0,1), (2,4), and (11,-2) is a right triangle. What is the area of the triangle?

8. Prove that the following points are vertices of a parallelogram:
   A: (-9,-5), B: (2,4), C: (-3,-3), and D: (-4,2).

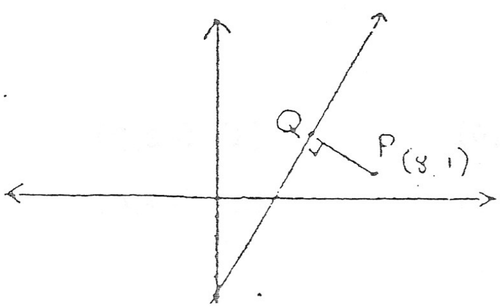
9. Write an equation showing P:(x,y) is equidistant from (3,9) and (2,12).

SECTION II — DISTANCE FROM A POINT TO A LINE

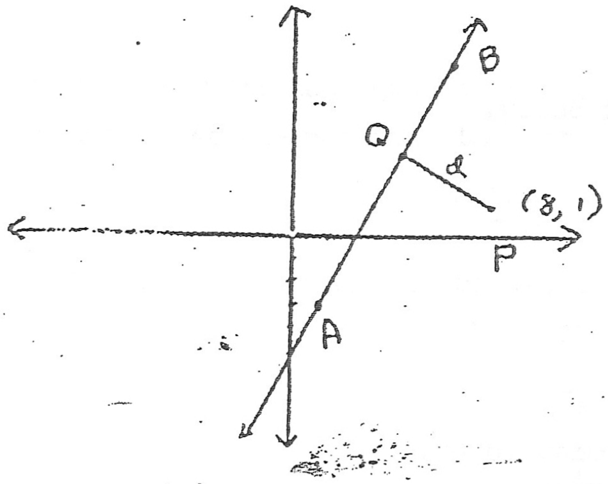
Actually, there are many distances from a point to a line. The Distance, the one recognized and talked about by mathematicians, is the shortest distance.

The shortest distance from a point to a line is the perpendicular distance.

The distance from point (8,1) to the line $y = 2x - 5$ is the length of the segment $\overline{PQ}$. (See diagram.)



Our job, in this section, is to determine the distance from a point to a line.



To determine the distance from P to the line we think along this way:

1. Find the slope of $\overleftrightarrow{PQ}$.
2. Find the equation of $\overleftrightarrow{PQ}$.
3. Find the coordinates of Q by solving the the systems of equations; i.e. the equations for $\overleftrightarrow{AB}$ and $\overleftrightarrow{PQ}$.
4. Finally use the distance formula to determine PQ.

1. Since $\overleftrightarrow{PQ}$ is perpendicular to $\overleftrightarrow{AB}$, the slope of PQ is $-\frac{1}{2}$.

2. Using the point (8,1) and the slope $-\frac{1}{2}$, write the equation for $\overleftrightarrow{PQ}$:

$$\frac{y-1}{x-8} = -\frac{1}{2} \qquad \text{This simplifies to:} \quad 2y = -x + 10.$$

3. Solve the system of equations: $y = 2x - 5$

$2y = -x + 10.$ The solution is (4,3).

(4,3) are the coordinates of point Q.

4. Find the distance between P and Q. This is: $\sqrt{(8-4)^2 + (3-1)^2} = \sqrt{20}$.

The above procedure is long. Mathematicians have a way of trying to generalize to create workable, simple formulas to speed up the tasks. Given a point P with coordinates (x_1, y_1) and a line with the equation $ax + by + c = 0$, and by following the four step procedure as outlined above, you can derive the formula for the distance from a point to a line.

The formula is:

$$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

<u>MEMORIZE THIS FORMULA</u>

Check out the point (8,1) and the line $y = 2x - 5$ with the new formula. Show that the formula also gives the distance as $\sqrt{20}$.

The following exercises deal with the use of the formula discussed in this section. Work at them carefully. At times you may be called upon to be somewhat creative. Don't let that bother you. Give it the old college try!

<u>EXERCISE 2</u>

1. Find the distance to the line $3x - 2y + 12 = 0$ from:

(a) (1,3) (b) (-1,7) (c) (0,0) (d) (-2,3)

(e) (2,-2) (f) (-3,-2) (g) (2,9) (h) (-4,4).

2. Find the distance from:

   (a) $(4,-1)$ to $4x + 3y = 6$ (b) $(-1,2)$ to $y = -3x - 8$

3. Find the distance between the parallel lines:

   (a) $2x + 7y + 5 = 0$ and $2x + 7y = 3$

   (b) $5x - 2y = 0$ and $5x - 2y = 8$.

4. Find the length of the shortest altitude of the triangle with vertices at $(5,3)$, $(1,-4)$, and $(-4,1)$.

5. Find the length of the shortest median of the triangle with vertices at $(4,-2)$, $(-1,-3)$, and $(3,2)$.

6. Determine the distance from the midpoint of the segment whose endpoints are $(4,6)$ and $(10,2)$ to the line $2x - 3y = -5$.

7. Find the equations of the two lines which are parallel to $3x + 4y - 5 = 0$, and at a distance 2 from it.

8. Find all points that are on the Y-axis and at a distance 4 from the line whose equation is $3x + 4y = 12$.

9. Find the equations of thetwo lines parallel to $x - 5y + 4 = 0$ and at a distance 3 from it.

10. $(1,4)$, $(4,9)$, $(8,10)$, and $(2,0)$ are vertices of a trapezoid. What is the length of the altitude of the trapezoid?

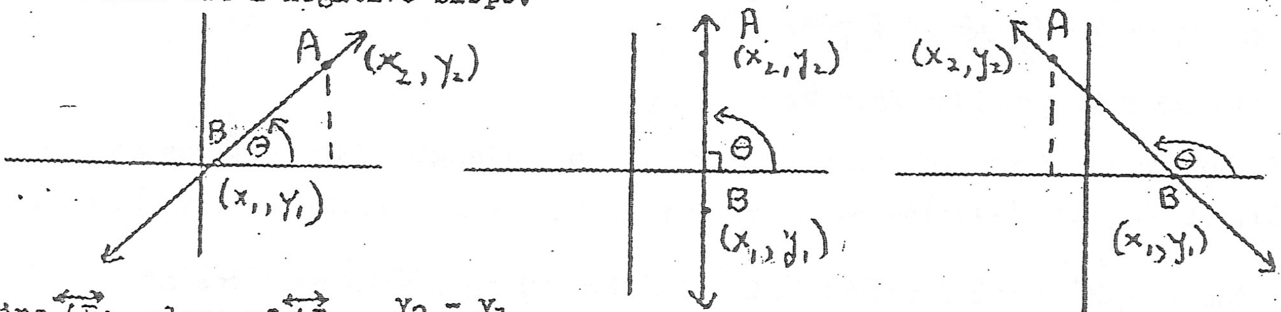
11. $(0,0)$, $(0,5)$, and $(3,4)$ are vertices of a rhombus. What are the coordinates of the fourth vertex? What is the length of the altitude of the rhombus?

12. Find the point on the line $6x - 5y = 3$ which is nearest the origin.

SECTION III

INCLINATION OF A LINE

In the diagrams below, θ is the angle of inclination of line $\overleftrightarrow{AB}$. The angle of inclination is the angle formed by the X-axis and the line. The angle is from the X-axis to the line in a counter-clockwise direction. If θ is acute the line has a positive slope. If $\theta = 90°$ the line is perpendicular to the X-axis. If θ is obtuse the line has a negative slope.



For line $\overleftrightarrow{AB}$: slope of $\overleftrightarrow{AB} = \dfrac{y_2 - y_1}{x_2 - x_1}$

For line $\overleftrightarrow{AB}$: $\tan\theta = \dfrac{y_2 - y_1}{x_2 - x_1}$

Note: The slope of $\overleftrightarrow{AB} = \tan\theta$. The slope of a line is equal to the tangent of its angle of inclination.

To find the inclination of a line:

1. Determine the slope of the line.
2. Use the tan table to find the measure of the angle of inclination.
3. If the slope is negative, use the table as though the slope were positive and then choose the supplement of the determined angle.

Example: Find the angle of inclination of the line whose equation is $y = 3x - 7$. The slope of the line is 3. Hence the measure of the angle of is approximately $72°$. Note: $\tan 72° = 3.0777$.
For the line $y = -3x$, the angle of inclination is approximately $108°$.

EXERCISE 3

1. Find the angle of inclination of each of the following lines. Sketch diagrams often.

(a) $y = x$ (b) $y = \frac{3}{4}x$ (c) $y = 2x$ (d) $y = -x$

(e) $y = -\frac{1}{4}x + 2$ (f) $y = 3x - 8$ (g) $3y + x = 7$

(h) $2y - 5x + 4 = 0$ (i) $y = 5$ (k) $x = 6$

2. Find the equation of a line whose angle of inclination is $60°$ and whose distance from the origin is 3. (Use $\tan 60° = \sqrt{3}$) (Two solutions.)

3. Write the equation of a line which is parallel to $5x + 12y = 26$ at a distance of 1 unit from the origin. (Two solutions.)

4. Determine the angle of inclination of the line which contains each of the following pairs of points:

(a) (2,7) and (4,9) (b) (2,7) and (2,11) (c) (4,4) and (7,4)

(d) (3,2) and (-4,7) (e) (-2,-3) and (-4,10)

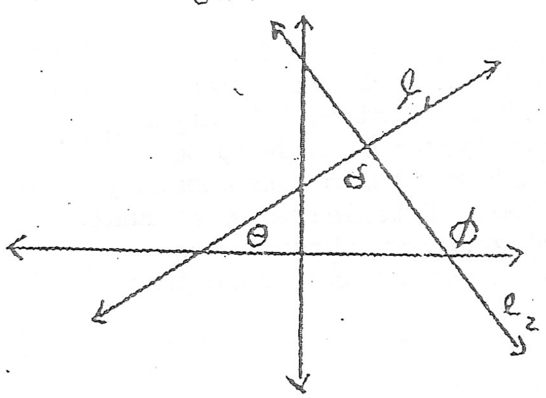
5. Write the equation of a line if the point nearest to the origin on the line is:.

(a) (3,3) (b) (2,3) (c) (-2,4)

SECTION IV THE ANGLE BETWEEN TWO INTERSECTING LINES

The notion of the angle between two intersecting lines is ambiguous. There are exactly four angles fromed by two intersecting lines. However, if the measure of one of these angles is found, it is a simple matter to determine the measures of the other three.

Study this diagram:



$\phi = \theta + \delta$ (The measure of an exterior angle of a triangle is equal to the sum of the two opposite interior angles.)

$\therefore \delta = \phi - \theta$

$\tan \delta = \tan(\phi - \theta)$

$\tan \delta = \dfrac{\tan \phi - \tan \theta}{1 + \tan \phi \tan \theta}$

Since $\tan \phi$ = the slope of ℓ_2 (m_2) and $\tan \theta$ = the slope of ℓ_1 (m_1)

The formula for the tan of the angle from ℓ_1 to ℓ_2 in terms of the slopes of the two lines is: $\tan \delta = \dfrac{m_2 - m_1}{1 + m_2 m_1}$

If δ is acute $\tan \delta$ will be positive. If δ is obtuse, $\tan \delta$ will be negative. If δ is a right angle, the formula fails. Get out your creative powers!

Should you forget the formula discussed above--or just plain don't like it--it is just a simple to find θ and ϕ, and then determine δ. $\delta = \phi - \theta$.

EXERCISE 4

1. Determine the measures of the angles between each pair of lines. Do not use the formulas. Use head and graphs only.

(a) $y = 3$ and $x = 7$ (b) $x = y$ and $x = 4$

(c) $y = 5$ and $x = y$ (d) $x = -y$ and $x = 7$

2. Find the acute angle formed by each of the following pairs of lines:

   (a) $y = 2x$ and $x = 3$ (b) $2x + 5y = 10$ and $y = 2$

   (c) $3x - 2y = 0$ and $2x + 3y = 6$ (d) $y = 3x + 2$ and $y = x - 4$

   (e) $x - 3y - 5 = 0$ and $x + y - 1 = 0$ (f) $x - 7y = 2$ and $4x - 3y = 6$

3. The vertices of a triangle are (0,0), (5,0) and (7,9). Find the angle between the two longest sides.

4. Find the equations of the lines through (2,3) making an angle of 45° with the line $2x - y + 3 = 0$.

5. Find the acute angle between the lines joining (2,1), (-3,-1) and (-1,2), (2,-2).

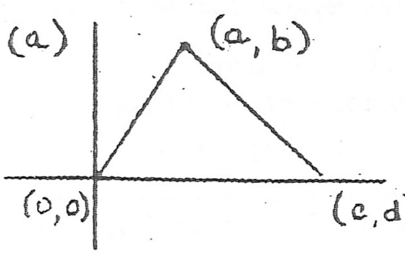
*SECTION V ANALYTIC PROOFS FOR PLANE GEOMETRY THEOREMS

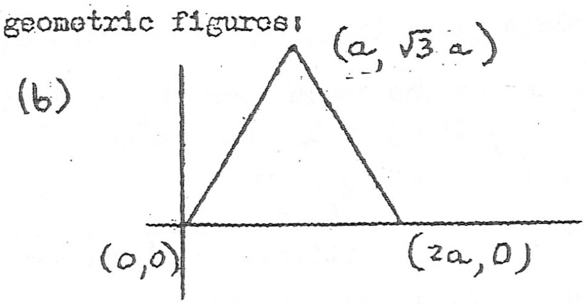
* This section for Math Analysis Students only.

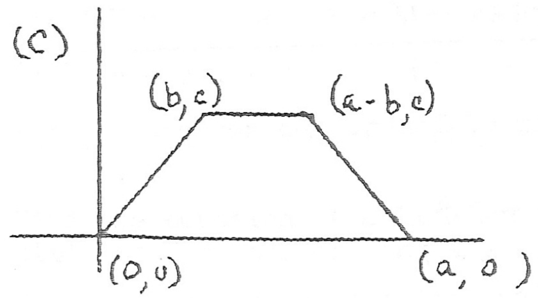
The properties of a given geometric figure usually found in Euclidean plane geometry do not in any way depend upon a related coordinate system. It often happens, however, that the introduction of a coordinate system will help to simplify the work of proving a theorem. The figure must be placed conveniently on the coordinate plane and generality must be preserved. This latter statement means--use letters for coordinates, not numbers! Wind your way through the following exercises. Check with your instructor on occasion to make sure that you are proving the theorems correctly.

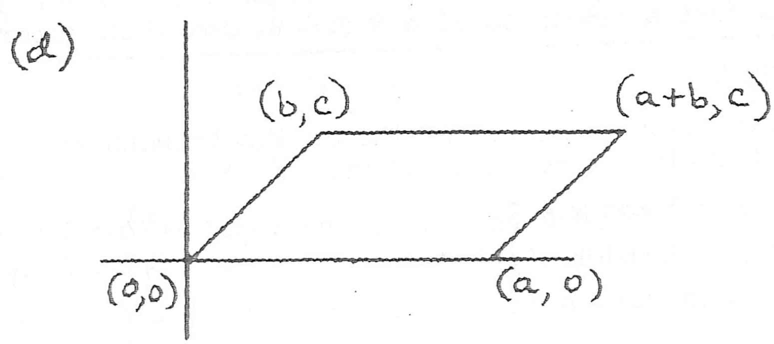
EXERCISE 5

1. Identify each of the following geometric figures:

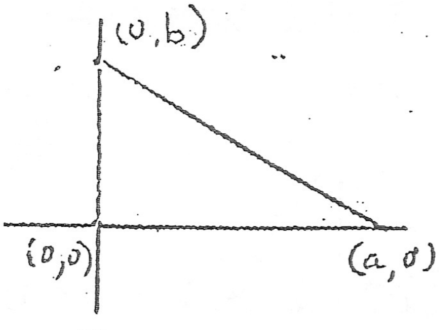




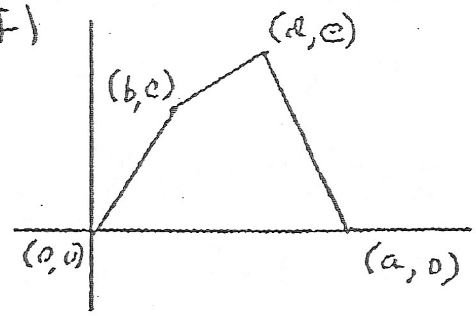




(e)



(f)



2. For each of the following: (a) Set up the figure on the coordinate system using coordinates as in exercise 1; (b) using methods of coordinate geometry prove the theorem.

   (a) The midpoint of the hypotenuse of a right triangle is equidistant from the vertices of the triangle. (When using midpoints label the triangle as in (e) above but use (2a,0) and (2b,0). This saves you the hassle of fractions.)

   (b) The diagonals of a rectangle are congruent.

   (c) The midpoints of the sides of any quadrilateral are the vertices of a parallelogram.

   (d) The distance between themidpoints of the nonparallel sides of a trapezoid is one-half the sum of the lengths of the parallel sides.

   (e) The sum of the squares of the lengths of the sides of a parallelogram equals the sum of the squares of the lengths diagonals.

   (f) In any triangle four times the sum of the squares of the lengths of the medians is equal to three times the sum of the squares of the lengths of the sides.

   (g) The diagonals of a rhombus are perpendicular.

   (h) The lines which join consecutively the midpoints of the sides of an isosceles trapezoid form a rhombus.

   (i) A quadrilateral is a parallelogram if the diagonals bisect each other.

SECTION VI. EVALUATION

1. Review the Behavioral Objectives.
2. Take the Trial Run.
3. Take the test.

   OR: Take a brief oral test on the high lights of this L.A.P. and do a paper on one of the following topics:

   1. Develop the basic formulas for point, line, plane, distance, etc. using a 3-dimensional coordinate system.

   2. Consider a 2-dimensional coordinate system where the Y-axis is at a 45° angle with the positive X-axis. Develop basic formulas for distance, line equations, parallelism, etc.

   3. Some creative project of your own. Get approval of your instructor

ANSWERS

1. (a) $2\sqrt{34}$; (b) $(5,-2)$; (c) $-\frac{5}{3}$; (d) $\frac{y+7}{x-8} = -\frac{5}{3}$ or $3y = -5x + 19$;

(e) $\frac{12}{5}$; (f) $\frac{19}{3}$; (g) $\frac{y-12}{x-4} = -\frac{5}{3}$ or $3y = -5x + 56$;

(h) $\frac{y-12}{x-4} = \frac{3}{5}$ or $5y = 3x + 48$

2. (a) 7; (b) 8; (c) $-\frac{8}{7}$; (d) $y = 7x$; (e) $\frac{y-2}{x-1} = -\frac{1}{7}$ or $7y = -x + 15$.

3. (a) $\frac{y-8}{x-3} = \frac{9}{5}$ or $5y = 9x + 13$; (b) $\frac{y-4}{x-1} = 2$ or $y = 2x + 2$;

(c) $y = 5$; (d) $y = -\frac{1}{2}x + 4$; (e) $y = -\frac{1}{2}x + 2$; (f) The points (4,0) and (0,2) are on the line. Hence the equation is: $\frac{y}{x-4} = -\frac{2}{4}$ or $2y = -x + 4$

(g) $x = 8$; (h) $\frac{y-2}{x} = \frac{7}{2}$ or $2y = 7x + 4$

4. $y = 4x + 17$; 5. (a) $x + 5y + 7 = 0$; (b) $-5x + y + 17 = 0$

6. Isosceles because two of the sides have a length of $\sqrt{10}$. The third side has length $\sqrt{20}$. $\sqrt{10}^{\,2} + \sqrt{10}^{\,2} = \sqrt{20}^{\,2}$. Hence the triangle is a right triangle. Area equals $\frac{1}{2}$ base $\cdot$ height. Area $= \frac{1}{2}\sqrt{10}\sqrt{10} = 5$ sq. units.

7. $\frac{\sqrt{1521}}{2}$ sq. units. 8. Use slopes. Show $\overline{AD} \parallel \overline{BC}$ and $\overline{AC} \parallel \overline{BD}$. The distance formula could also be used. A quadrilateral is a parallelogram if both pairs of opposite sides are congruent. 9. $(x-3)^2 + (y-9)^2 = (x-2)^2 + (y-12)^2$ simplified, #9 becomes: $3y = x + 29$. Notice that all points P lie on the perpendicular bisector of the segment determined by the two given points.

EXERCISE 2

1. (a) $\frac{9}{\sqrt{13}}$; (b) $\frac{5}{\sqrt{13}}$; (c) $\frac{12}{\sqrt{13}}$; (d) 0; (e) $\frac{22}{\sqrt{13}}$; (f) $\frac{7}{\sqrt{13}}$; (g) 0

(h) $\frac{8}{\sqrt{13}}$; 2. (a) $\frac{7}{5}$; (b) $\frac{7}{\sqrt{10}}$; 3. (a) Choose a point on the first line such as $(1,-1)$; distance is $\frac{8}{\sqrt{53}}$; (b) $\frac{8}{\sqrt{29}}$.

4. Sketch a diagram--observe the shortest of the three altitudes. Find the distance from $(4,-2)$ to the line determined by $(1,-\frac{1}{2})$ and $(3,2)$; $\frac{55}{\sqrt{85}}$

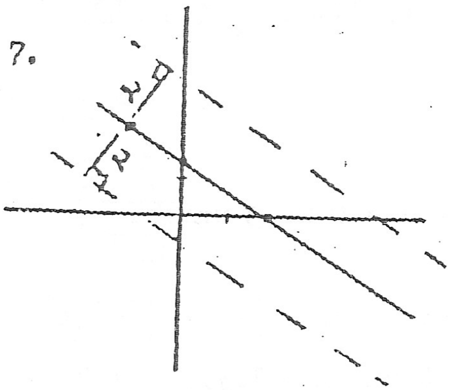
5. A median is a segment from a vertex to the midpoint of the opposite side. $\frac{\sqrt{145}}{2}$

Lines--Answers continued

<u>Exercise 2</u>

6. $\frac{7}{\sqrt{13}}$;

7.



The two lines are parallel so the equations of the lines will be of the form $3x + 4y + c = 0$.

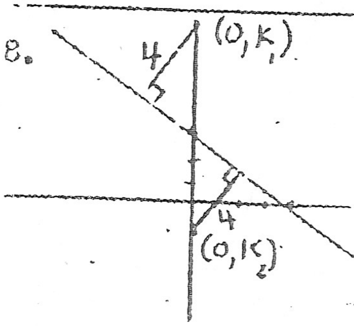
Choose a point on the given line such as (-1,2)

$$2 = \frac{|-3 + 8 + c|}{5}$$

Solutions: $3x + 4y + 5 = 0$

$3x + 4y - 15 = 0$

8.



$$4 = \frac{|4k - 12|}{5}$$

$k = 8$ or $k = -2$

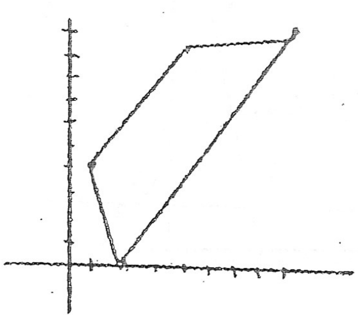
Solution: (0,8), (0,-2)

9. Method as in #7

Solution: $x - 5y + 3\sqrt{26} + 4 = 0$

$x - 5y + 4 - 3\sqrt{26} = 0$

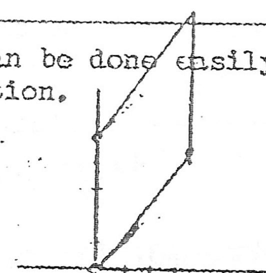
10.



Distance from (1,4) to the line determined by (2,0) and (8,10)

Solution: $\frac{17}{\sqrt{34}}$

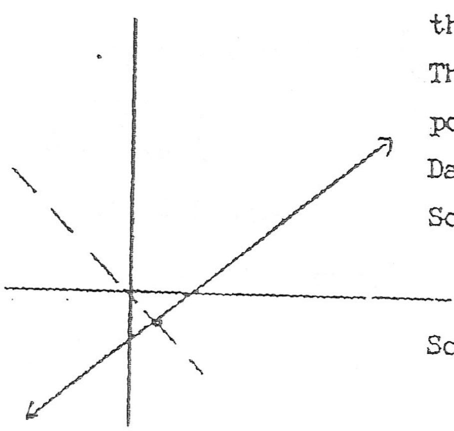
11. This can be done easily by inspection.



Fourth point: (3,9)

Altitude: 3 units.

12. The distance from a point to a line is the perpendicular.



The line contains the point (0,0).

Dashed line: $6y = -5x$.

Solve the system: $5x + 6y = 0$

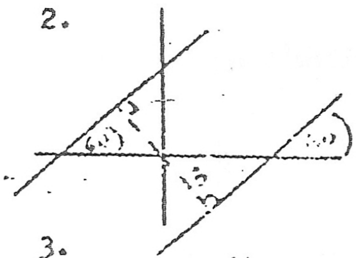
$6x - 5y = 3$ to find the point of intersection.

Solution: $(\frac{18}{61}, -\frac{15}{61})$

<u>Exercise 3</u> (Most answers approximate)

1. (a) 45°; (b) 37°; (c) 63.4°; (d) 135°; (e) 166°; (f) 72°;
(g) 162°; (h) 68°; (i) 0°; (j) 90°

2.

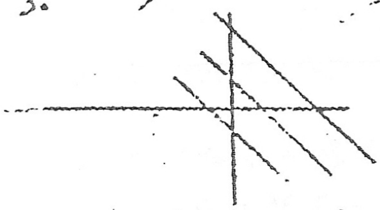


$\tan 60^\circ = \sqrt{3}$

From (0,0) to $-\sqrt{3}\,x + y + c = 0$ is 3 units.

Solution: $-\sqrt{3}x + y \pm 6 = 0$

3.



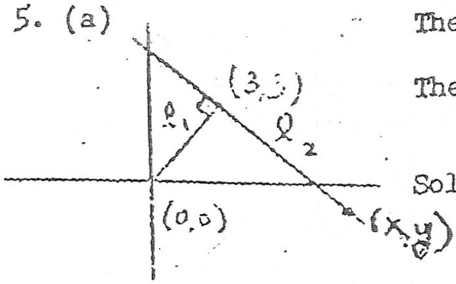
New lines will be of the form $5x + 12y + c = 0$

Distance from (0,0) to the new lines is 1.

Solution: $5x + 12y \pm 13 = 0$

4. (a) 45°; (b) 90°; (c) 0°; (d) 144.5°; (e) 99°.

5. (a)



The slope of ℓ_1 is 1

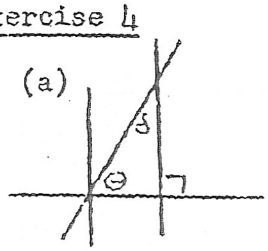
The slope of ℓ_2 is -1

Solution: $\frac{y-3}{x-3} = -1$ or $y = -x + 6$

(b) $3y = -2x - 13$; (c) $2y = x + 10$

<u>Exercise 4</u>

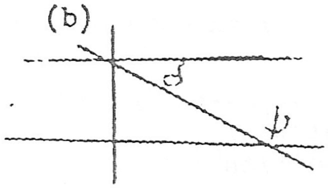
1. (a)



δ and θ are complementary

$\theta = 63.4^\circ$ hence, $\delta = 26.6^\circ$

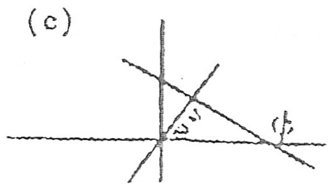
(b)



δ and $\emptyset$ are supplementary.

$\delta = 21.8^\circ$

(c)



$\delta = \emptyset - \theta$; $\tan \delta = \frac{\tan \emptyset - \tan \theta}{1 + \tan \emptyset \tan \theta}$

$\delta = 90^\circ$ (Notice, too, the slopes are negative reciprocals)

(d) $\tan \phi = \tan (\theta - \quad)$

$\tan \phi = \frac{3 - 1}{1 + 3} = \frac{1}{2}$

$\phi = \underline{26.6^\circ}$

(e) $\tan \phi = \frac{3 + 1}{1 - 3} = -2$

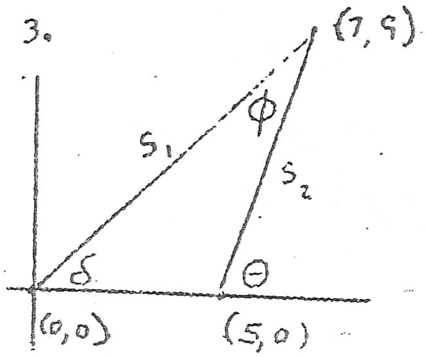
$\phi = 116.6^\circ$

The acute angle is $\underline{63.4^\circ}$

(f) $\tan \phi = \dfrac{\frac{1}{7} - \frac{4}{3}}{1 + \frac{4}{21}}$

$= \frac{3 - 28}{21 + 4} = \frac{-25}{25} = -1.$ $\phi = 135.$ Hence, the acute angle is $\underline{45^\circ}$.

3.



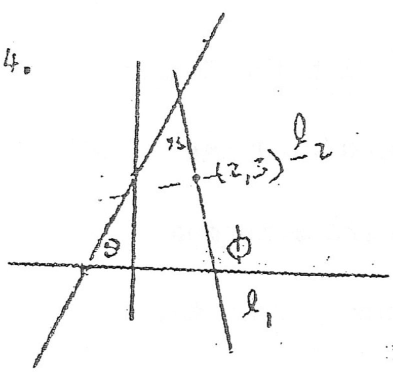
By inspection, the longest sides are determined by (0,0)--(7,9)--s_1; and (5,0)--(7,9)--s_2

slope of s_1 is 9/7; slope of s_2 is 9/2

$\tan \phi = \frac{9/2 - 9/7}{1 + 81/14} = \frac{63 - 18}{14 + 81} = \frac{45}{95} = .4737$

$\phi = \underline{25^\circ}$

4.



$\tan \theta = 2$; $\tan \delta = 1$

$\phi = \theta + \delta$

$\tan \phi = \frac{2 + 1}{1 - 2} = -3.$ The slope of ℓ_1 is -3

Equation of ℓ_1: $\frac{y - 3}{x - 2} = -3$ or $y = -3x + 9$

ℓ_2 is perpendicular to ℓ_1. Hence, the equation of ℓ_2 is: $\frac{y - 3}{x - 2} = \frac{1}{3}$

or: $3y = x + 7$

5. The slope of ℓ_1 is $\frac{2}{5}$. The slope of ℓ_2 is $-\frac{4}{3}$. The acute angle has measure 75°

Exercise 5

1. (a) Triangle; (b) Equilateral triangle; (c) Trapezoid;

(d) Parallelogram; (e) Right triangle; (f) Quadrilateral

MATH ANALYSIS-ANALYTIC GEOMETRY LINES--TRIAL RUN

1. For each pair of points given determine:

(a) The equation of the line containing them.
(b) The distance between them.
(c) The midpoint of the segment they determine.
(d) The slope of the line they determine.
(e) The Y-intercept of the line they determine.
(f) The X-intercept of the line they determine.
(g) The inclination of the line they determine.

1. (3,9) and (5,3) 2. (10,0) and (-4,-9) * 3. (a,b) and (c,d)

2. Write the equation of the line which contains the point (7,-3) and is

(a) parallel to the line which has the equation $4x - 7y = 8$.
(b) perpendicular to the line which has the equation $4x - 7y = 8$.

3. Find the distance between

(a) The point (2,3) and the line $5x = 2y - 7$.
(b) The lines $8x - 2y + 5 = 0$ and $8x = 2y$.
(c) The lines $y = 4$ and $y = 8$.

4. For each pair of lines graph and determine the angles at their point of intersection.

(a) $y = 7$ and $x = 3y - 2$ (b) $x = 7$ and $x = y$.
(c) $5x - 3y = 8$ and $2x - 6y = 0$

5. Solve each of the following:

(a) What is the equation of a line that makes an angle of 150° with the X-axis and is 1 unit from the origin. (Two solutions)

(b) Write the equation of a line whose angle of inclination is 60° and the line is 12 units from the origin.

(c) Find the coordinates of P if P is the point on $4x - 2y = 3$ and P is the point on the line which is nearest to the origin.

(d) Find all points which lie on the Y-axis and are at a distance 3 from the line which has the equation $3x + 4y = 12$.

(e) Find the equation of the line which passes through (3,2) and the intersection of the lines $2x - y = 3$ and $3x - 2y = 7$.

6. Math Analysis students review the coordinate geometry proofs in the last section of the L.A.P.

* Math Analysis students only.

It is impossible not to feel stirred at the thought of the emotions of men at certain historic moments of adventure and discovery--Columbus when he first saw the Western shore, Pizarro when he stared at the Pacific Ocean, Franklin when the electric spark came from the string of his kite, Galileo when he first turned his telescope to the heavens. Such moments are also granted to students in the abstract regions of thought, and high among them must be placed the morning when Descartes lay in bed and invented the method of co-ordinate geometry.

Alfred North Whitehead

LINE L.A.P. TRIAL RUN ANSWERS

1. 1. (a) $y = -3x + 18$
   (b) $\sqrt{40}$
   (c) (4,6)
   (d) -3
   (e) 18
   (f) 6
   (g) 108.4°

2. (a) $14y = 9x - 90$
   (b) $\sqrt{277}$
   (c) (3, -9/2)
   (d) 9/14
   (e) -45/7
   (f) 10
   (g) 32.7°

3. (a) $\frac{y - b}{x - a} = \frac{b - d}{a - c}$

   (b) $\sqrt{(a - c)^2 + (b - d)^2}$

   (c) $\left(\frac{a + c}{2}, \frac{b + d}{2} \right)$

   (d) $\frac{b - d}{a - c}$

   (e) $\frac{-a(b - d)}{a - c} + b$

   (f) $\frac{-b(a - c)}{b - d} + a$

   (g) arc tan $\frac{b - d}{a - c}$

2. (a) $4x - 7y = 49$
   (b) $7x + 4y = 37$

3. (a) $\frac{11}{\sqrt{29}}$; (b) $\frac{5}{\sqrt{68}}$; (c) 4.

4. (a) 71.6° and 108.4°; (b) 45° and 135°; (c) 40.6° and 139.4°

5. (a) $x - \sqrt{3}\, y \pm 2 = 0$; (b) $-\sqrt{3}\, x + y \pm 24 = 0$; (c) $\left(\frac{3}{5}, -\frac{3}{10}\right)$;
   (d) $(0, \frac{27}{4})$ and $(0, -\frac{3}{4})$; (e) $4y = 7x - 13$

Socrates used to meditate all day in the snow, but Descartes' mind only worked when he was warm. Bertrand Russell