THE WRAPPING FUNCTION

Learning Activities Package

BEHAVIORAL OBJECTIVES

I. Define

A. The domain of the wrapping function
B. The range of the wrapping function
C. The unit circle
D. The reference angle

II. Given a length in terms of π determine

A. The measure of the corresponding central reference angle
B. Its corresponding range element using the wrapping function

III. Convert

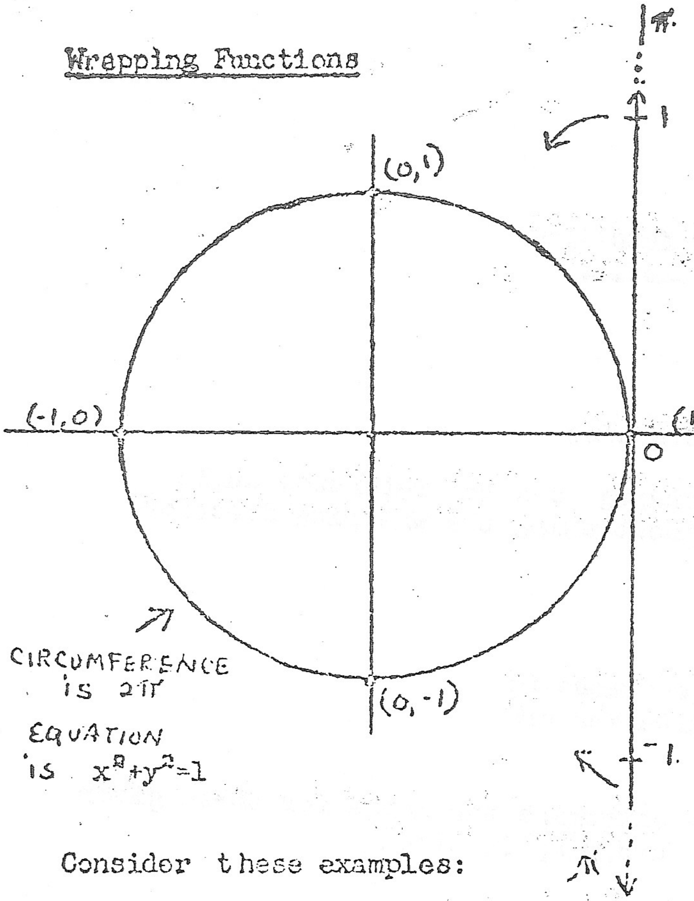
A. A radian measure to a degree measure
B. A degree measure to a radian measure

IV. Use the Pythagorian theorem to determine the range for these given elements of the domain of the wrapping function

A. $\frac{\pi}{6}$ B. $\frac{\pi}{3}$ C. $\frac{\pi}{4}$

V. Given the wrapping function determine its

A. Domain
B. Range
C. Period



For the wrapping function we consider the real number line and the unit circle as shown in this diagram. The unit circle can be defined as either

1. The circle whose equation is $x^2 + y^2 = 1$

or

2. A circle with its center at the origin and radius of 1.

A number is selected on the number line. The number line is then wrapped around the unit circle. The selected real number is the domain element. The point it touches on the unit circle is the corresponding range element. This point is given by an ordered pair. (x,y) such that $x^2+y^2=1$ The circumference of the circle is 2π.

Consider these examples:

(1) For domain element π, (select the point π or the number line and wrap) The point π on the number line lands at the coordinate $(-1, 0)$.

(2) If W is the wrapping function then: [$W(\frac{\pi}{2})$ is read the wrap of $\frac{\pi}{2}$]

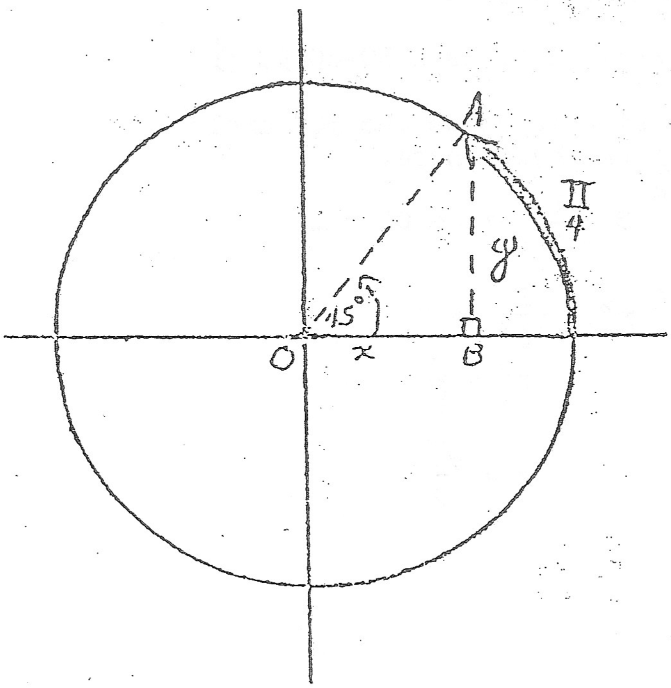
$W(\frac{\pi}{2}) = (0,1)$

$W(-\frac{\pi}{2}) = (0,-1)$

$W(2\pi) = (1,0)$

$W(\frac{3\pi}{2}) = (0,-1)$

$W(-\pi) = (-1,0)$



How about $W(\frac{\pi}{4})$?

$m(\angle AOB) = 45°$

Hence $\triangle AOB$ is an isosceles right triangle. Thus $\overline{OB} \cong \overline{AB}$.

Since the circle is a unit circle $OA = 1$

By the Pythagorian theorem $(OB)^2 + (AB)^2 = 1^2$

OR

$x^2 + y^2 = 1$

$x^2 + x^2 = 1$ [Since $x = y$]

$2x^2 = 1$

$x^2 = \frac{1}{2}$

$x = \sqrt{\frac{2}{2}} = y$

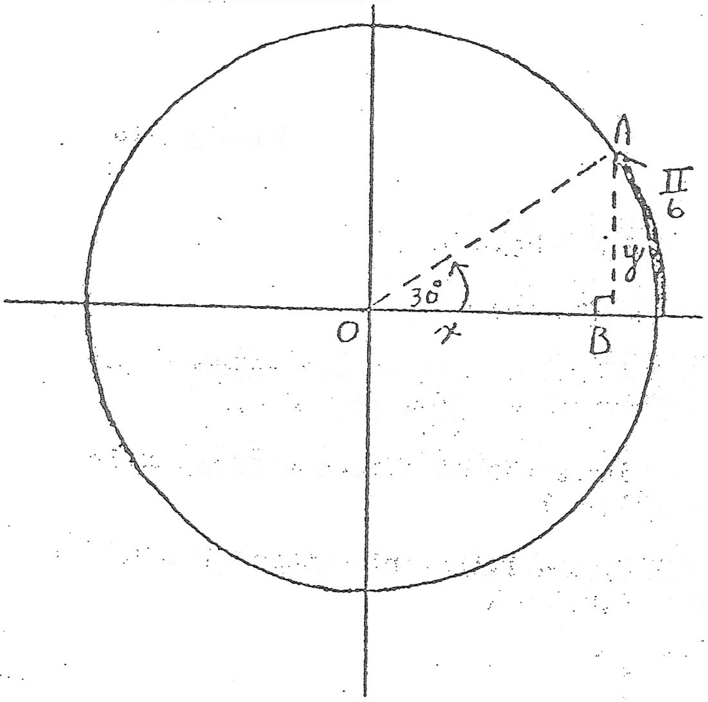
Consequently the coordinates of A are $(\frac{\sqrt{2}}{2}, \sqrt{\frac{2}{2}})$

So $W(\frac{\pi}{4}) = (\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$

ASSIGNMENT 1:

Determine:

1. $W(\frac{3\pi}{4}) =$ ________
2. $W(-\frac{\pi}{4}) =$ ________
3. $W(-\frac{3\pi}{4}) =$ ________
4. $W(\frac{5\pi}{4}) =$ ________
5. $W(\frac{9\pi}{4}) =$ ________
6. $W(\frac{11\pi}{4}) =$ ________
7. $W(-\frac{5\pi}{4}) =$ ________
8. $W(\frac{29\pi}{4}) =$ ________



$W(\frac{\pi}{6}) = ?$

$m(\angle AOB) = 30^\circ$

Triangle A O B is a 30-60-90 $\triangle$.

The side opposite the 30° angle is $\frac{1}{2}$ the hypotenuse.

So, $AB = \frac{1}{2}$; since $OA = 1$

Now:

$$x^2 + (\frac{1}{2})^2 = 1$$

$$x^2 = \frac{3}{4}$$

$$x = \frac{\sqrt{3}}{2}$$

Consequently the coordinates of A are $(\frac{\sqrt{3}}{2}, \frac{1}{2})$

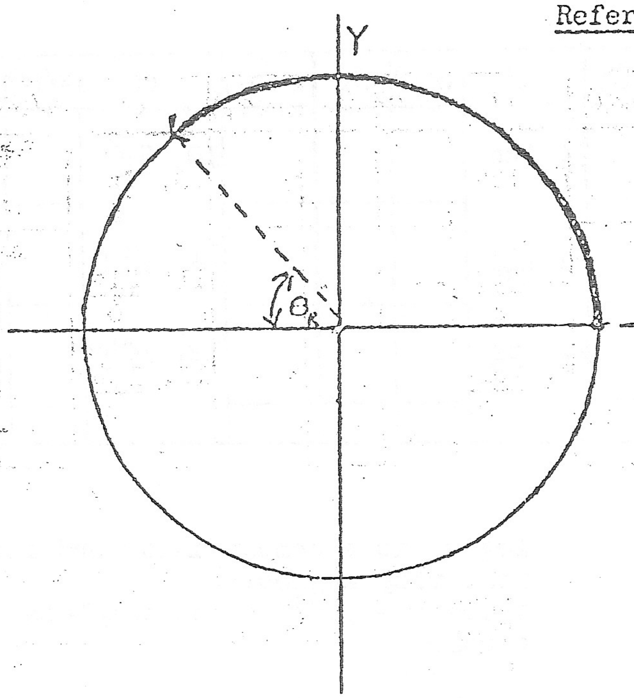
So $W(\frac{\pi}{6}) = (\frac{\sqrt{3}}{2}, \frac{1}{2})$

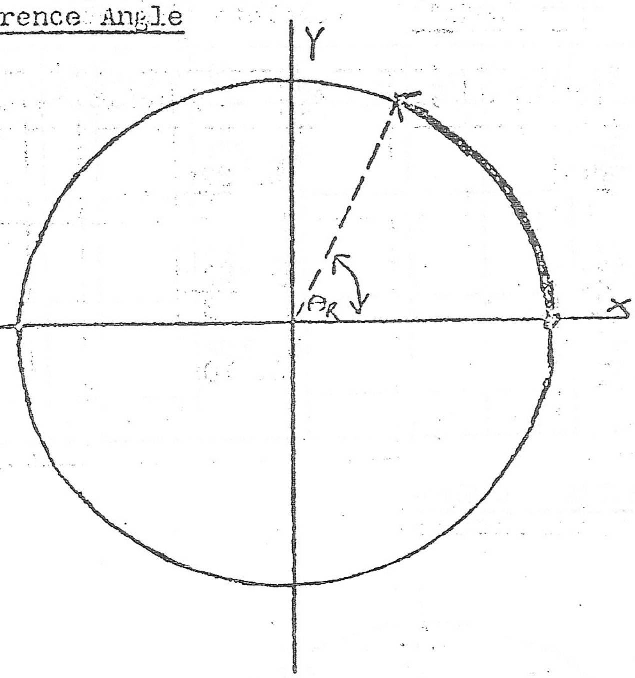
ASSIGNMENT 2:

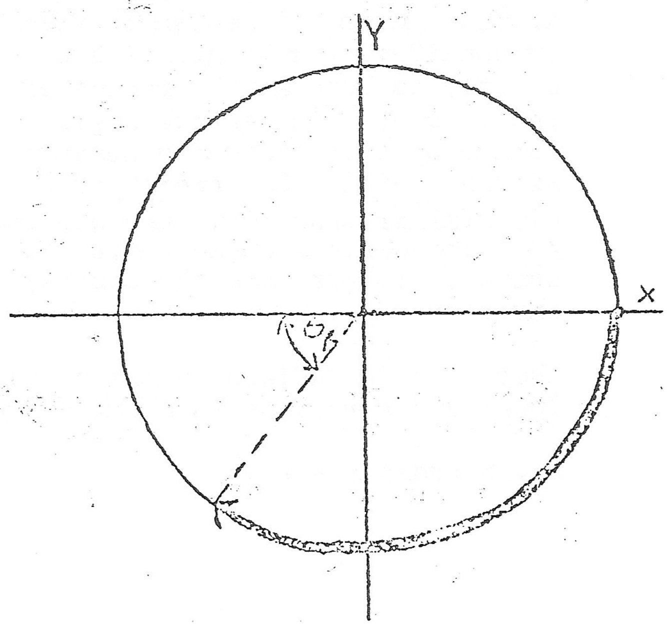
Determine:

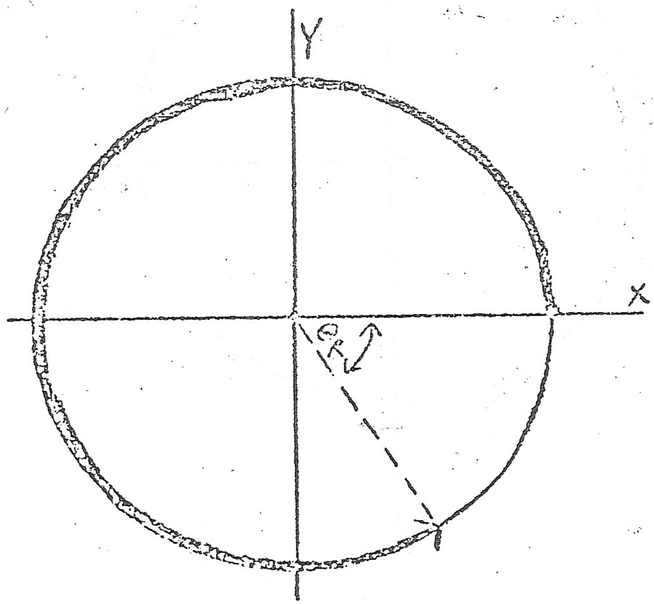
1. $W(\frac{5\pi}{6}) =$ ______________
2. $W(-\frac{\pi}{6}) =$ ______________
3. $W(\frac{7\pi}{6}) =$ ______________
4. $W(-\frac{5\pi}{6}) =$ ______________
5. $W(\frac{11\pi}{6}) =$ ______________
6. $W(\frac{13\pi}{6}) =$ ______________
7. $W(\frac{\pi}{3}) =$ ______________
8. $W(\frac{4\pi}{3}) =$ ______________
9. $W(\frac{5\pi}{3}) =$ ______________
10. $W(-\frac{\pi}{3}) =$ ______________
11. $W(-\frac{4\pi}{3}) =$ ______________
12. $W(\frac{7\pi}{3}) =$ ______________

Reference Angle









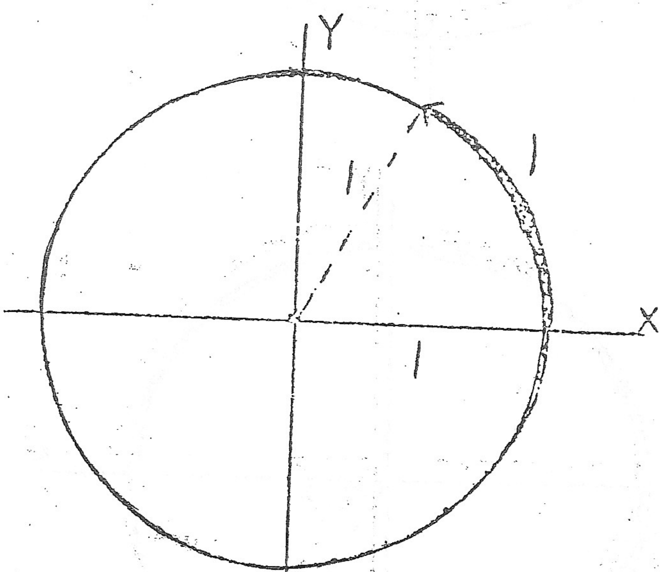
✱ Notice $W(\theta)$ and $W(\theta_R)$ have the same coordinates except for sign. Thus it is really only important to learn the coordinates for the first quadrant well. The sign can easily be supplied by observing the quadrant which contains the endpoint of $W(\theta)$.

The reference angle is the angle θ_R formed by a radius to the specific domain element on the unit circle and the X-axis. $0^\circ \leq \theta_R \leq 90^\circ$. Note, θ_R is never larger than 90°.

ASSIGNMENT 3: Complete the table.

θ	$W(\theta)$	θ_R	$W(\theta_R)$	θ	$W(\theta)$	θ_R	$W(\theta_R)$	θ	$W(\theta)$	θ_R	$W(\theta_R)$	θ	$W(\theta)$	θ_R	$W(\theta_R)$
1. $\frac{8\pi}{3}$				4. $\frac{29\pi}{4}$				7. 18π				10. $\frac{7\pi}{6}$			
2. $\frac{3\pi}{4}$				5. $\frac{87\pi}{3}$				8. $\frac{11\pi}{2}$				11. $\frac{21\pi}{6}$			
3. $\frac{\pi}{2}$				6. 10π				9. $\frac{-61\pi}{4}$				12. $\frac{104\pi}{3}$			

RADIAN MEASURE



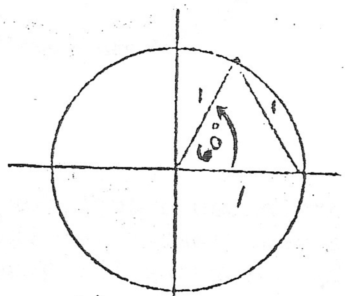
(1)

Degree is a common unit word for measuring angles. There are 360° in one complete circle.

Another unit for measuring angles is the radian. A radian is formed by considering a length of arc exactly as long as the radius of the circle. The central angle formed by this arc has a measure of one radian. One radian is a bit smaller than 60°. See diagram #2. The central angle in (2) is somewhat longer than the central angle in (1).

There are 2π radians in one circle. There are π radians in a semi circle.

Or, π radians = 180°.



(2)

ASSIGNMENT 4: Complete the following table showing radian – degree relationships.

RADIANS	DEGREES	RADIANS	DEGREES	RADIANS	DEGREES
$\frac{\pi}{2}$		$\frac{\pi}{6}$		* 2	
3π		$\frac{5\pi}{6}$			50°
$\frac{\pi}{4}$		$\frac{7\pi}{6}$			90°
$\frac{\pi}{6}$		$\frac{21\pi}{4}$			10°
$\frac{3\pi}{4}$		$\frac{2\pi}{3}$			150°
8π		$\frac{\pi}{7}$			728°
0		2.8π			16°
$\frac{3\pi}{2}$		* 3			12°
$\frac{3\pi}{4}$		* 1			130°

HINT: Use Proportion

$$* \quad \frac{\pi \text{ radians}}{180^\circ} = \frac{X \text{ radians}}{y^\circ} \quad \text{or} \quad X \text{ radians} = \frac{y^\circ \cdot \pi \text{ radians}}{180^\circ}$$

PROPERTIES OF THE WRAPPING FUNCTION

1. The domain of the wrapping function is ______________________

2. The range of the wrapping function is ______________________

3. The wrapping function is periodic. This means that the range elements repeat at a regular interval. The period for the wrapping function is 2π.

NOTE: $W(0) = W(2\pi) = W(4\pi) = \ldots = W(2k\pi)$

$W(\frac{\pi}{4}) = W(\frac{9\pi}{4}) = W(\frac{17\pi}{4}) = \ldots W(\frac{\pi}{4} + 2k\pi)$ or $W(\frac{(8k + 1)\pi}{4})$

ASSIGNMENT 5: Complete this table. Generalize in the last column.

$W(\frac{\pi}{2}) = W(\quad) = W(\quad) = \ldots = W(\quad)$

$W(\frac{3\pi}{2}) = W(\quad) = W(\quad) = \ldots = W(\quad)$

$W(\pi) = W(\quad) = W(\quad) = \ldots = W(\quad)$

$W(\frac{5\pi}{6}) = W(\quad) = W(\quad) = \ldots = W(\quad)$

$W(-\frac{\pi}{4}) = W(\quad) = W(\quad) = \ldots = W(\quad)$

$W(-\frac{3\pi}{4}) = W(\quad) = W(\quad) = \ldots = W(\quad)$

EVALUATION

1. Review the L.A.P.
2. Take the Trial Run
3. Take the Test

TRIAL RUN

1. What is the domain of the Wrapping Function?
2. What is the range of the Wrapping Function?
3. What is the equation of the unit circle?
4. What is the period of the Wrapping Function?
5. Complete the following table using the Wrapping Function.

DOMAIN θ	$W(\theta)$	θ_R	$W(\theta+\pi)$	$W(\theta-\frac{\pi}{2})$	$W(\theta+2\pi)$
0					
$\frac{\pi}{2}$					
$-\frac{3\pi}{4}$					
$\frac{10\pi}{3}$					
$-\pi$					
$\frac{\pi}{6}$					
$-\frac{5\pi}{6}$					
$-\frac{7\pi}{6}$					
$\frac{2\pi}{3}$					
$\frac{28\pi}{3}$					
-400π					
$-\frac{\pi}{3}$					

6. Complete this table:

DEGREE MEASUREMENT	RADIAN MEASURE
90°	
	$\frac{3\pi}{4}$
	2π
	1
	π
-45°	
720°	
	3
	3π
30°	
60°	
-120°	
165°	

7. Each circle contains ______ radians.

8. In a circle of radius 4, an arc of length 8 subtends ______ radians.

9. Using the fact that the wrapping function is periodic complete this statement:

$W(\frac{3\pi}{4}) = W(\quad) = W(\quad) =$

$W(\qquad)$

↑

Generalization

ANSWER SHEET

ASSIGNMENT 1.

1. $\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$
2. $\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$
3. $\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$
4. $\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$
5. $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$
6. $\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$
7. $\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$
8. $\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

ASSIGNMENT 2.

1. $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
2. $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$
3. $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$
4. $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$
5. $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$
6. $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
7. $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
8. $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$
9. $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$
10. $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$
11. $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
12. $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

ASSIGNMENT 3.

1.	$\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$	60° or $\frac{\pi}{3}$	$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
2.	$\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$	45° or $\frac{\pi}{4}$	$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$
3.	(0, 1)	90° or $\frac{\pi}{2}$	(0, 1)
4.	$\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$	45° or $\frac{\pi}{4}$	$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$
5.	(-1, 0)	0°	(1,0)
6.	(1, 0)	0°	(1, 0)
7.	(1, 0)	0°	(1, 0)
8.	(0, -1)	90° o $\frac{\pi}{2}$	(0, 1)
9.	$\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$	45° or $\frac{\pi}{4}$	$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$
10.	$\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$	30°	$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
11.	(0, -1)	90° or $\frac{\pi}{2}$	(0, 1)
12.	$\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$	60° or $\frac{\pi}{3}$	$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

ASSIGNMENT 4

Degrees		Radious	Degrees
90°	30°		114.59°
540°	150°	$\frac{5\pi}{18}$	
45°	210°	$\frac{\pi}{2}$	
30°	945°	$\frac{\pi}{18}$	
135°	120°	$\frac{5\pi}{6}$	
1440°	$25\frac{5}{7}^\circ$	$\frac{182\pi}{45}$	
0°	504°	$\frac{4\pi}{45}$	
270°	171.89°	$\frac{\pi}{15}$	
135°	57.3°	$-\frac{13\pi}{18}$	

ASSIGNMENT 5

$$W\left(\frac{5\pi}{2}\right) = W\left(\frac{9\pi}{2}\right) = \ldots = W\left[\frac{(4k+1)\pi}{2}\right]$$

$$W\left(\frac{7\pi}{2}\right) = W\left(\frac{11\pi}{2}\right) = \ldots = W\left[\frac{(4k+3)\pi}{2}\right]$$

$$W\,(3\pi) = W\,(5\pi) = \ldots = W\left[(2k+1)\pi\right]$$

$$W\left(\frac{17\pi}{6}\right) = W\left(\frac{29\pi}{6}\right) = \ldots = W\left[\frac{(12k+5)\pi}{6}\right]$$

$$W\left(-\frac{9\pi}{4}\right) = W\left(\frac{-17\pi}{4}\right) = \ldots = W\left[\frac{-(8k+1)\pi}{4}\right]$$

$$W\left(\frac{-11\pi}{4}\right) = W\left(\frac{-19\pi}{4}\right) = \ldots = W\left[\frac{-(8k+3)\pi}{4}\right]$$

Trial Run Answers

1. D: {real numbers}
2. R: $\{(x,y): x^2+y^2 = 1\}$
3. $x^2+y^2 = 1$
4. 2π

5)

0	(1,0)	0	(-1,0)	(0,-1)	(1,0)
$\frac{\pi}{2}$	(0,1)	90°	(0,-1)	(1, 0)	(0,1)
$\frac{-3\pi}{4}$	$\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$	45°	$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$	$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$	$\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$
$\frac{10\pi}{3}$	$\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$	60°	$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$	$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$	$\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
$-\pi$	(-1,0)	0°	(1,0)	(0,1)	(-1,0)
$\frac{\pi}{6}$	$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$	30°	$\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$	$\left(+\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$	$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
$\frac{-5\pi}{6}$	$\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$	30°	$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$	$\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$	$\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$
$\frac{-7\pi}{6}$	$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$	30°	$\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$	$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$	$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
$\frac{2\pi}{3}$	$\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$	60°	$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$	$\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$	$\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
$\frac{28\pi}{3}$	$\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$	60°	$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$	$\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$	$\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$
-400π	(1,0)	0	(-1,0)	(0,-1)	(1,0)
$\frac{-\pi}{3}$	$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$	60°	$\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$	$\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$	$\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

Degree measure	Radian measure
	$\frac{\pi}{2}$
135°	
360°	
57.3°	
180°	
	$-\frac{\pi}{4}$
	4π
171.9°	
540°	
	$\frac{\pi}{6}$
	$\frac{\pi}{3}$
	$\frac{-2\pi}{3}$
	$\frac{11\pi}{12}$

7. 2π

8. 2

9. $\frac{11\pi}{4}$ $\quad \frac{19\pi}{4}$ $\quad \frac{(8K+3)\pi}{4}$